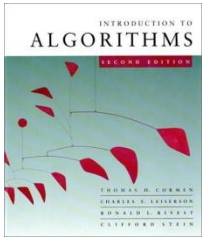
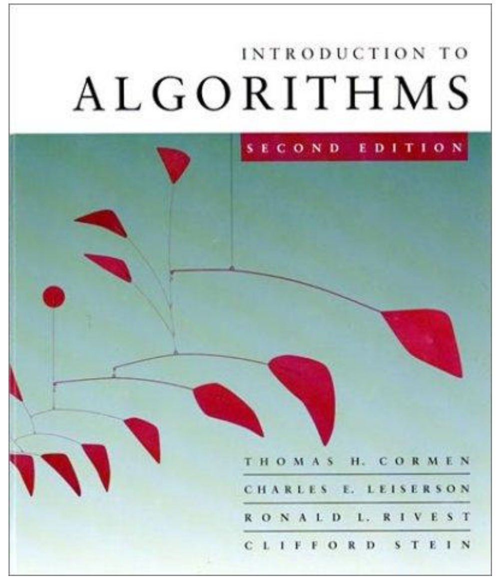


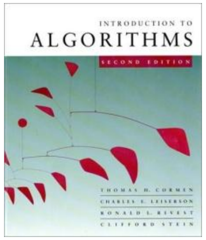
CS 5633 – Spring 2012



Single Source Shortest Paths

Carola Wenk

Slides courtesy of Charles Leiserson with small changes by Carola Wenk

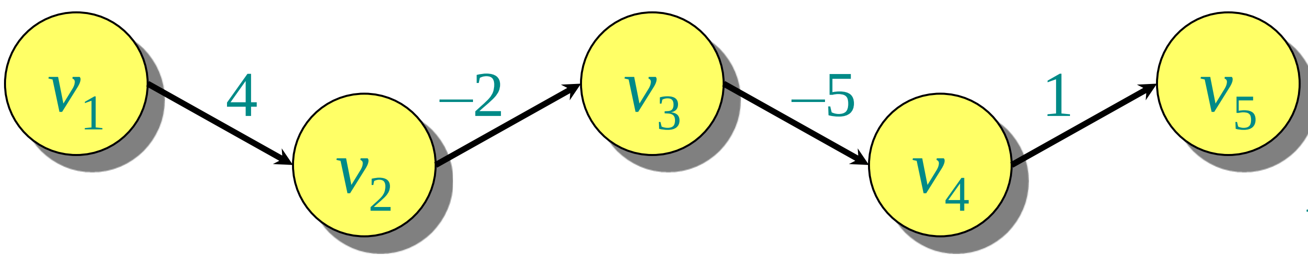


Paths in graphs

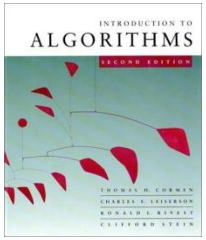
Consider a digraph $G = (V, E)$ with edge-weight function $w : E \to \mathbb{R}$. The ***weight*** of path $p = v_1 \to v_2 \to \ldots \to v_k$ is defined to be

$$w(p) = \sum_{i=1}^{k-1} w(v_i, v_{i+1}).$$

Example:



$w(p) = -2$

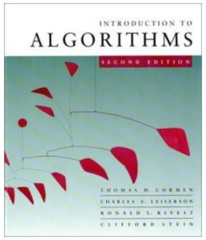


Shortest paths

A ***shortest path*** from u to v is a path of minimum weight from u to v. The ***shortest-path weight*** from u to v is defined as

$\delta(u, v) = \min\{w(p) : p$ is a path from u to $v\}$.

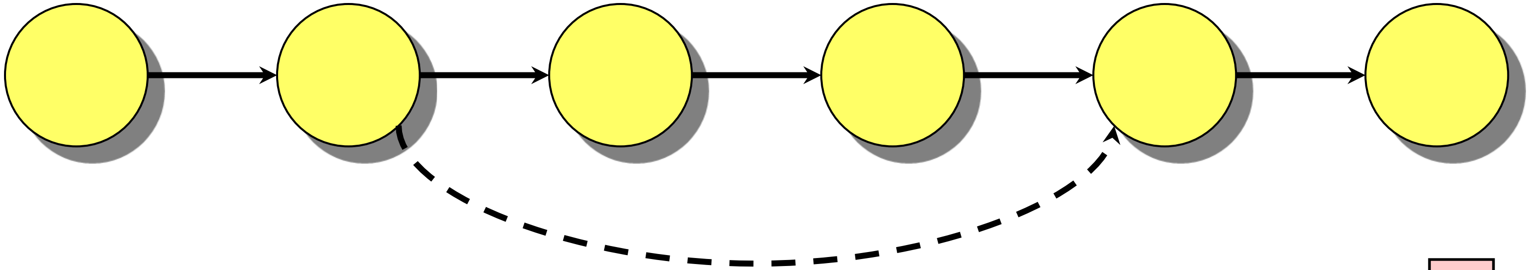
Note: $\delta(u, v) = \infty$ if no path from u to v exists.

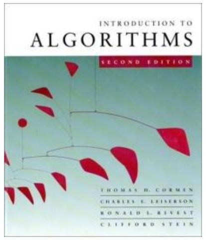


Optimal substructure

Theorem. A subpath of a shortest path is a shortest path.

Proof. Cut and paste:





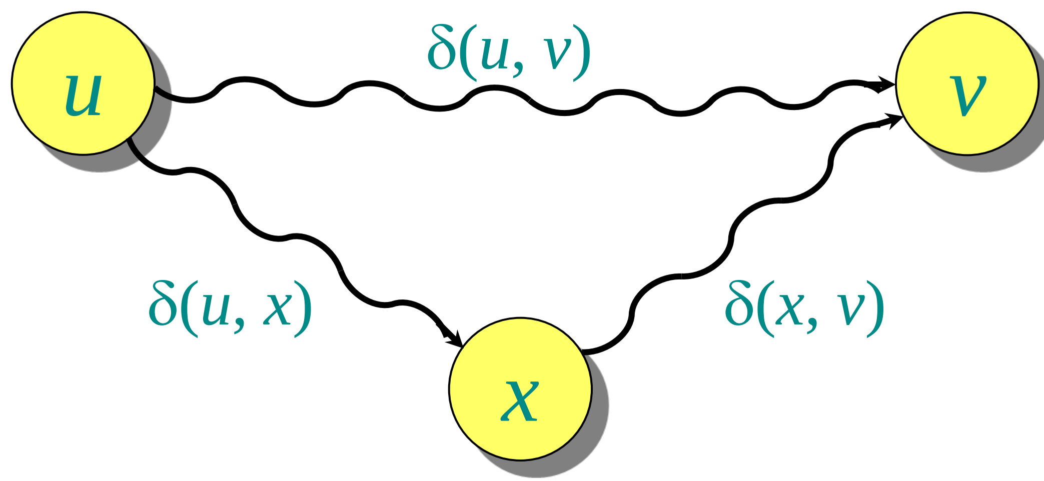
Triangle inequality

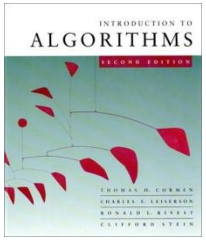
Theorem. For all $u, v, x \in V$, we have

$$\delta(u, v) \le \delta(u, x) + \delta(x, v).$$

Proof.

- $\delta(u,v)$ minimizes over **all** paths from u to v
- Concatenating two shortest paths from u to x and from x to v yields **one** specific path from u to v

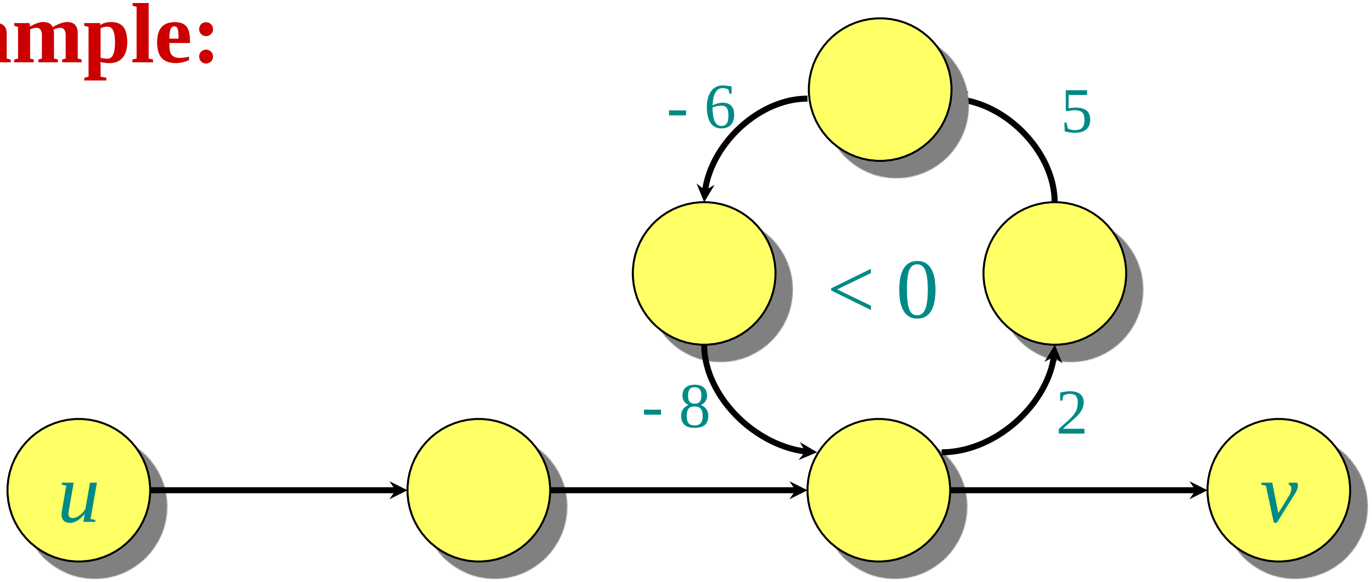


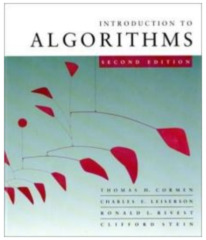


Well-definedness of shortest paths

If a graph *G* contains a negative-weight cycle, then some shortest paths may not exist.

Example:





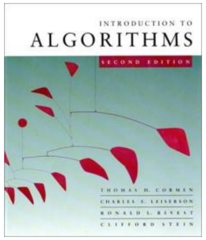
Single-source shortest paths

Problem. From a given source vertex $s \in V$, find the shortest-path weights $\delta(s, v)$ for all $v \in V$.

Assumption: All edge weights $w(u, v)$ are ***nonnegative***. It follows that all shortest-path weights must exist.

IDEA: Greedy.

1. Maintain a set S of vertices whose shortest-path weights from s are known, i.e., $d[v]=d(s,v)$
2. At each step add to S the vertex $v \in V - S$ whose distance estimate from s is minimal.
3. Update the distance estimates of vertices adjacent to v.



Dijkstra's algorithm

$d[s] \leftarrow 0$

for each $v \in V - \{s\}$

  do $d[v] \leftarrow \infty$

$S \leftarrow \varnothing$ ⊳ Vertices for which $d[v]=d(s,v)$

$Q \leftarrow V$ ⊳ Q is a priority queue maintaining $V - S$

while $Q \neq \varnothing$ **do**

  $u \leftarrow$ EXTRACT-MIN(Q)

  $S \leftarrow S \cup \{u\}$

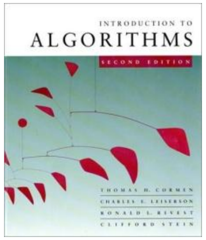
  for each $v \in Adj[u]$ **do**

    if $d[v] > d[u] + w(u, v)$ **then**

      $d[v] \leftarrow d[u] + w(u, v)$

relaxation step

Implicit DECREASE-KEY



Dijkstra

$d[s] \leftarrow 0$

for each $v \in V - \{s\}$

  do $d[v] \leftarrow \infty$

$S \leftarrow \varnothing$ ⊳ Verti

$Q \leftarrow V$ ⊳ Q is

while $Q \neq \varnothing$ **do**

  $u \leftarrow$ EXTRACT-MIN(Q)

  $S \leftarrow S \cup \{u\}$

  for each $v \in Adj[u]$ **do**

    if $d[v] > d[u] + w(u, v)$ **then**

      $d[v] \leftarrow d[u] + w(u, v)$

Implicit DECREASE-KEY

PRIM's algorithm

$Q \leftarrow V$

$key[v] \leftarrow \infty$ for all $v \in V$

$key[s] \leftarrow 0$ for some arbitrary $s \in V$

while $Q \neq \varnothing$

  do $u \leftarrow$ EXTRACT-MIN(Q)

    for each $v \in Adj[u]$

      do if $v \in Q$ and $w(u, v) < key[v]$

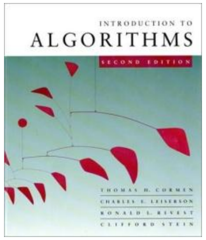
        then $key[v] \leftarrow w(u, v)$

          $\pi[v] \leftarrow u$

Difference to Prim's:

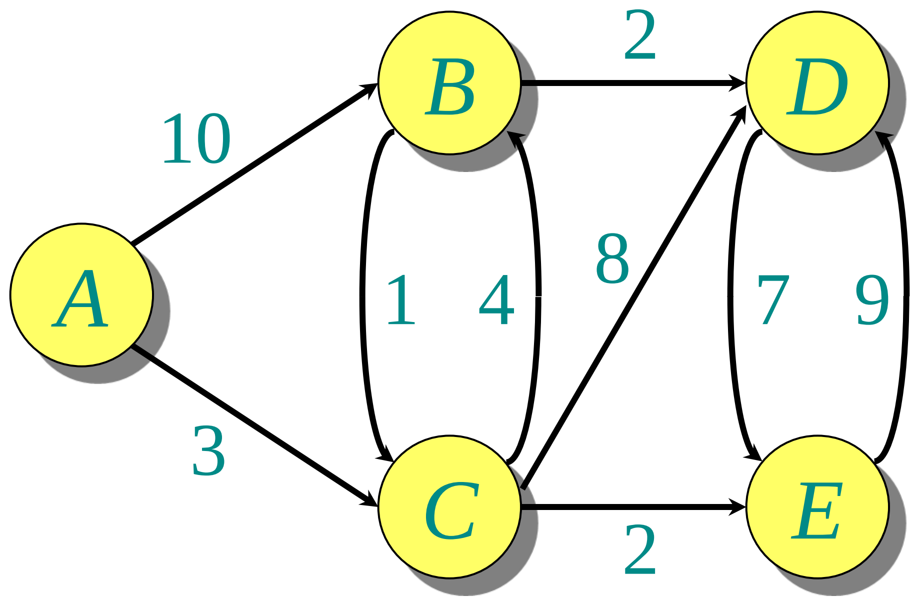
- It suffices to only check $v \in Q$, but it doesn't hurt to check all v
- Add $d[u]$ to the weight

relaxation step

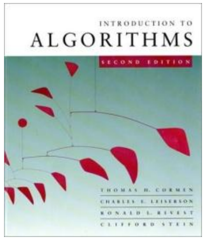


Example of Dijkstra's algorithm

Graph with nonnegative edge weights:

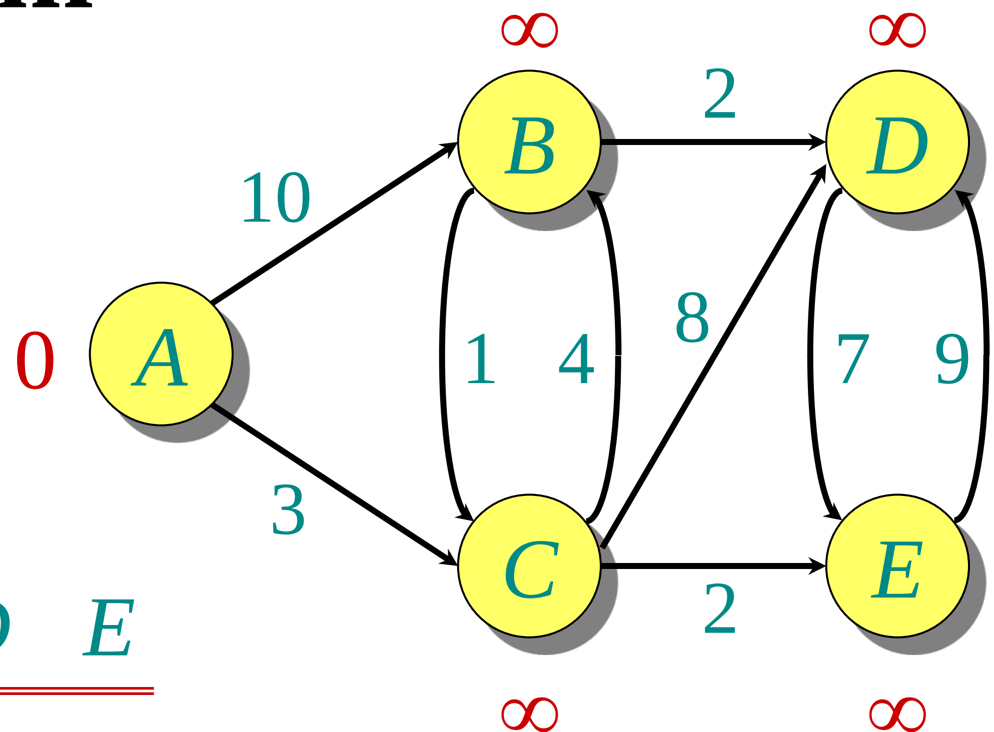


while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

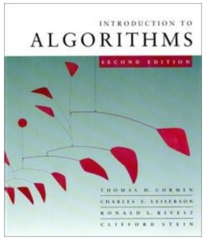
Initialize:



S: { }

Q:	A	B	C	D	E
	0	∞	∞	∞	∞

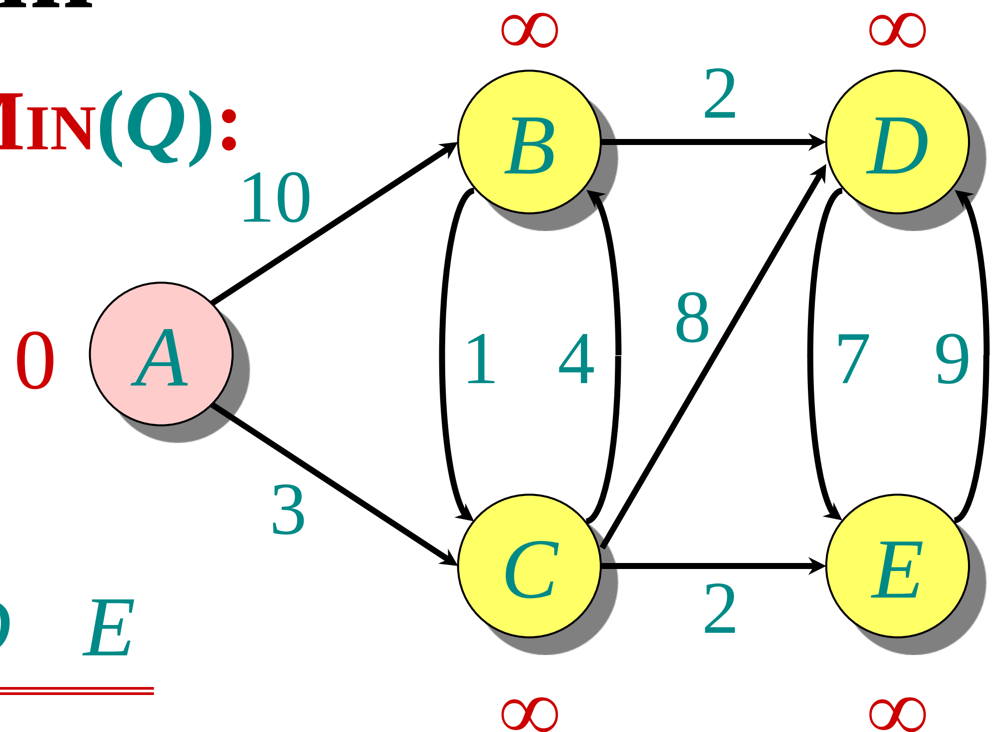
while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

"A" ← EXTRACT-MIN(Q):

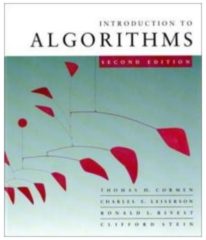
S: { A }



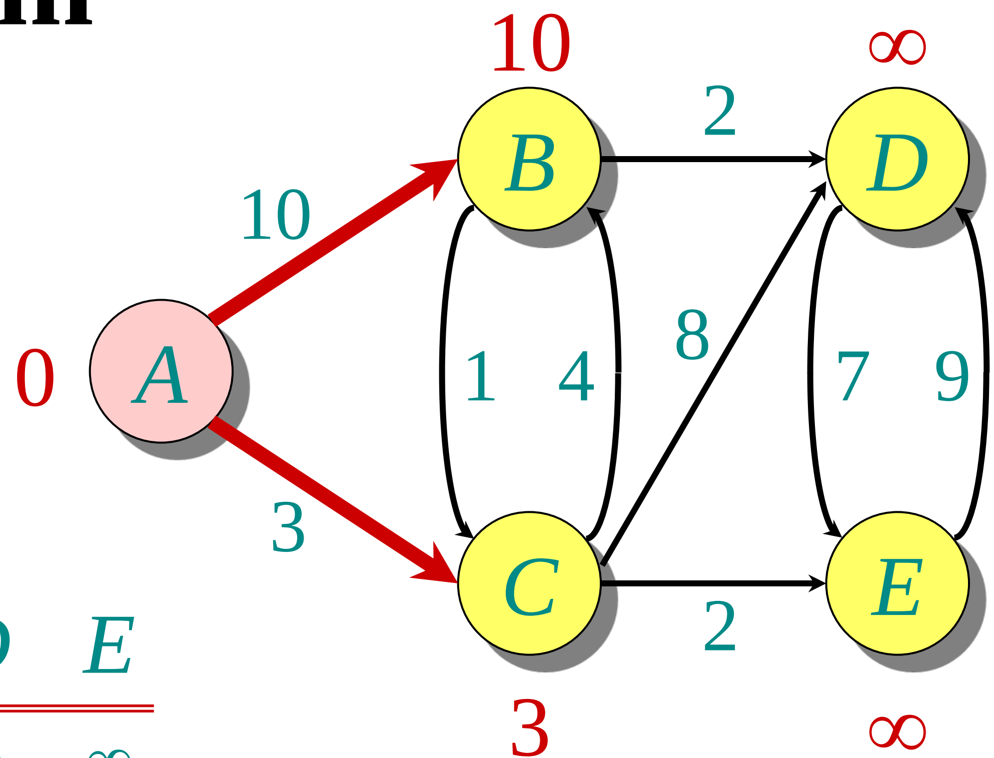
Q:

A	B	C	D	E
0	∞	∞	∞	∞

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

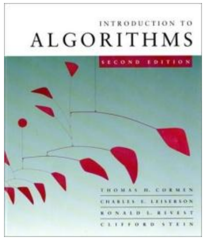


Relax all edges leaving *A*:

S: { A }

Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	–	–

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$

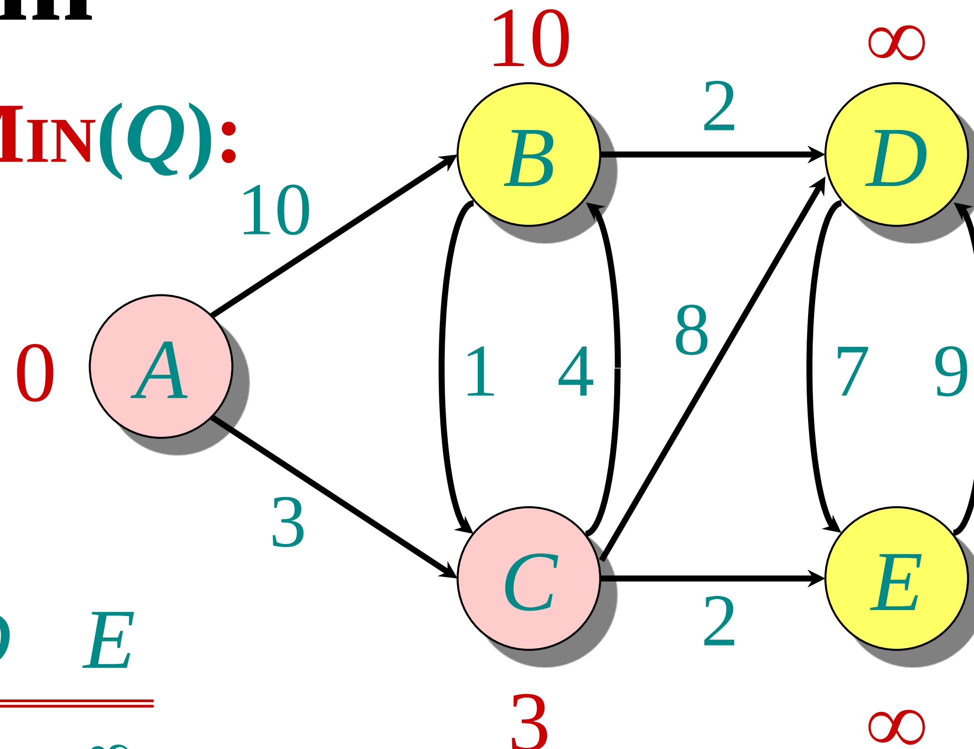


Example of Dijkstra's algorithm

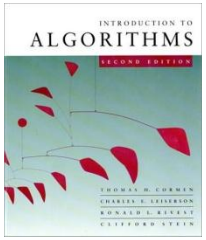
"C" $\leftarrow$ EXTRACT-MIN(Q):

S: { A, C }

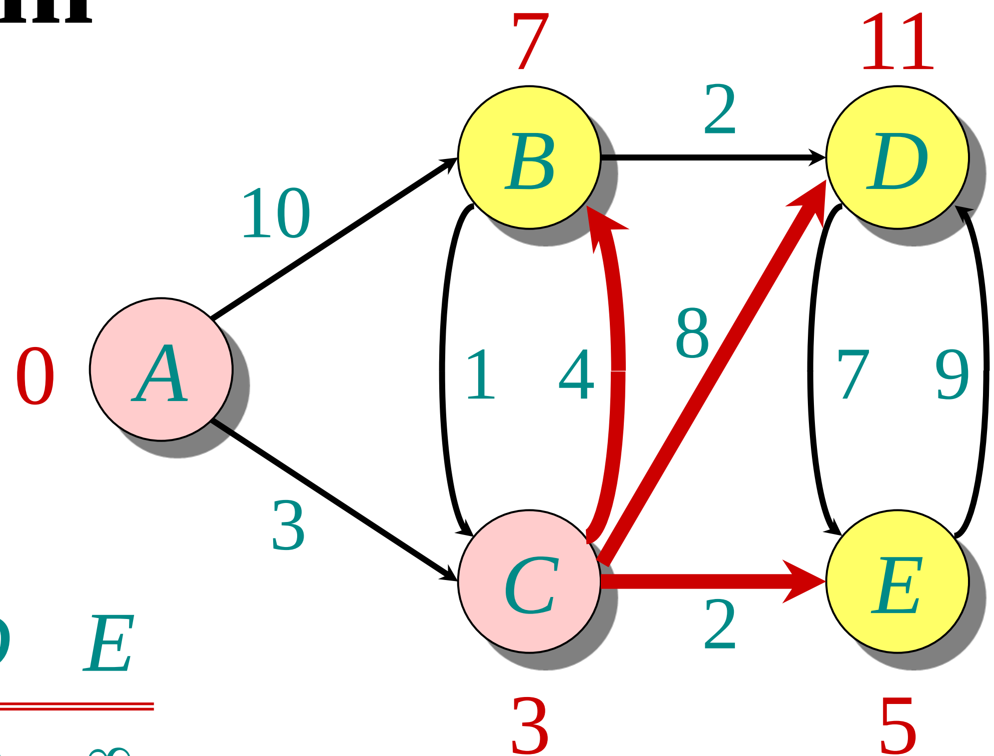
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	–	–



while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

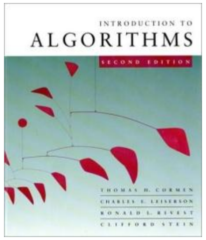


Relax all edges leaving *C*:

S: { *A*, *C* }

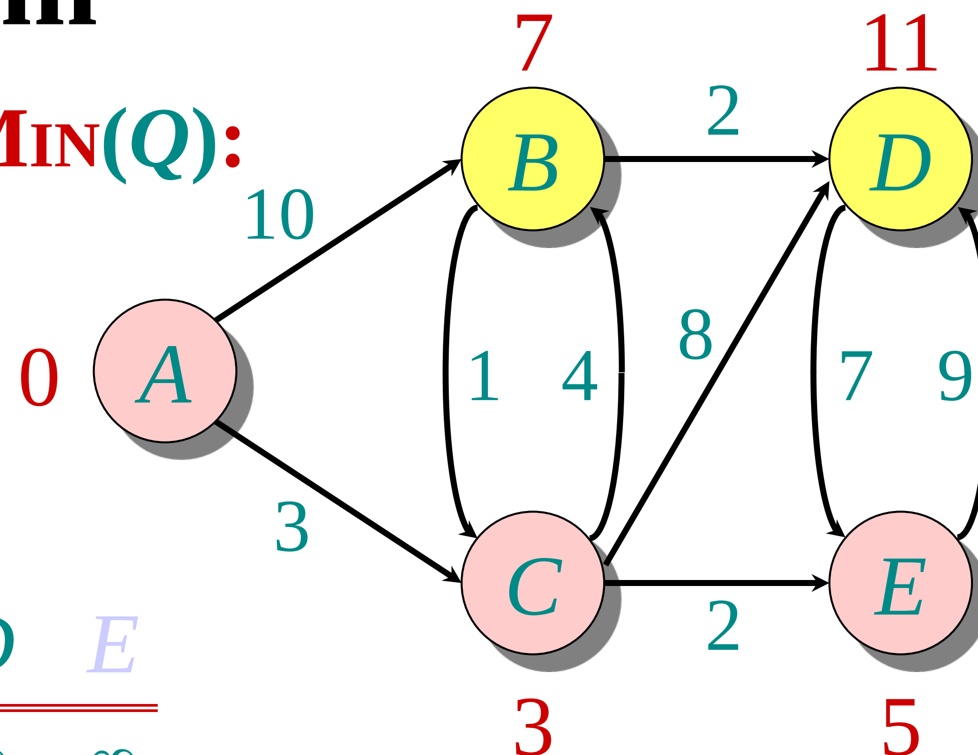
Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	–	–
		7		11	5

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

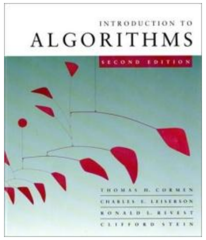
"*E*" ← EXTRACT-MIN(*Q*):



S: { *A*, *C*, *E* }

Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	–	–
		7		11	5

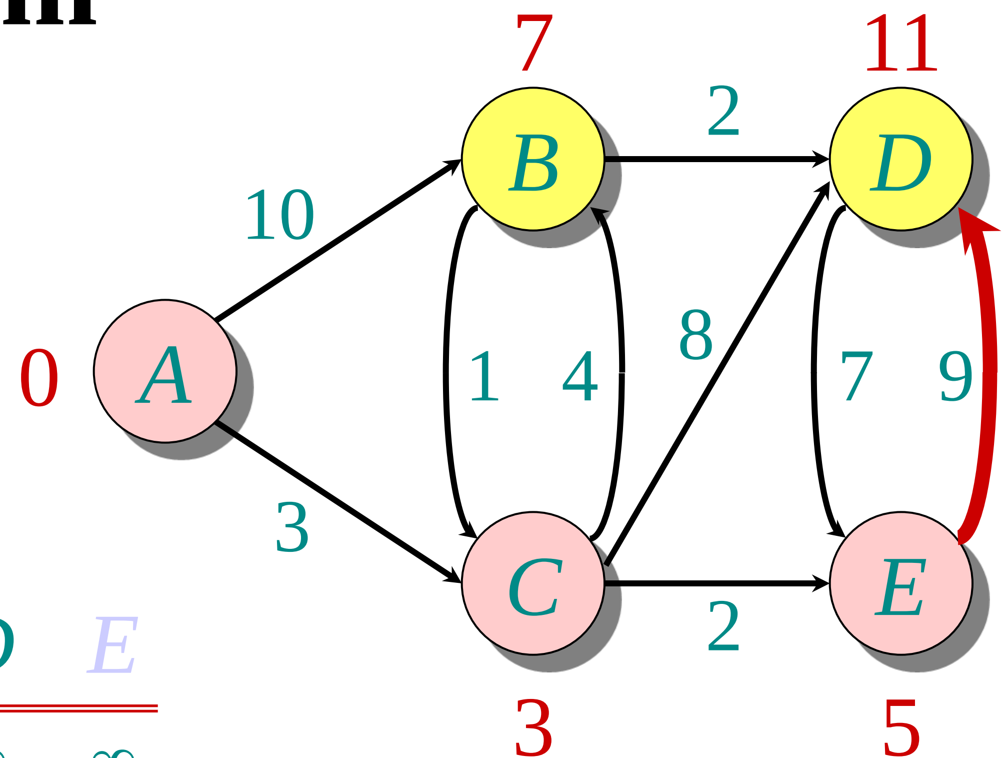
while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

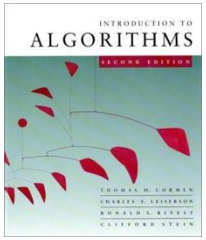
Relax all edges leaving *E*:

S: { *A*, *C*, *E* }



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

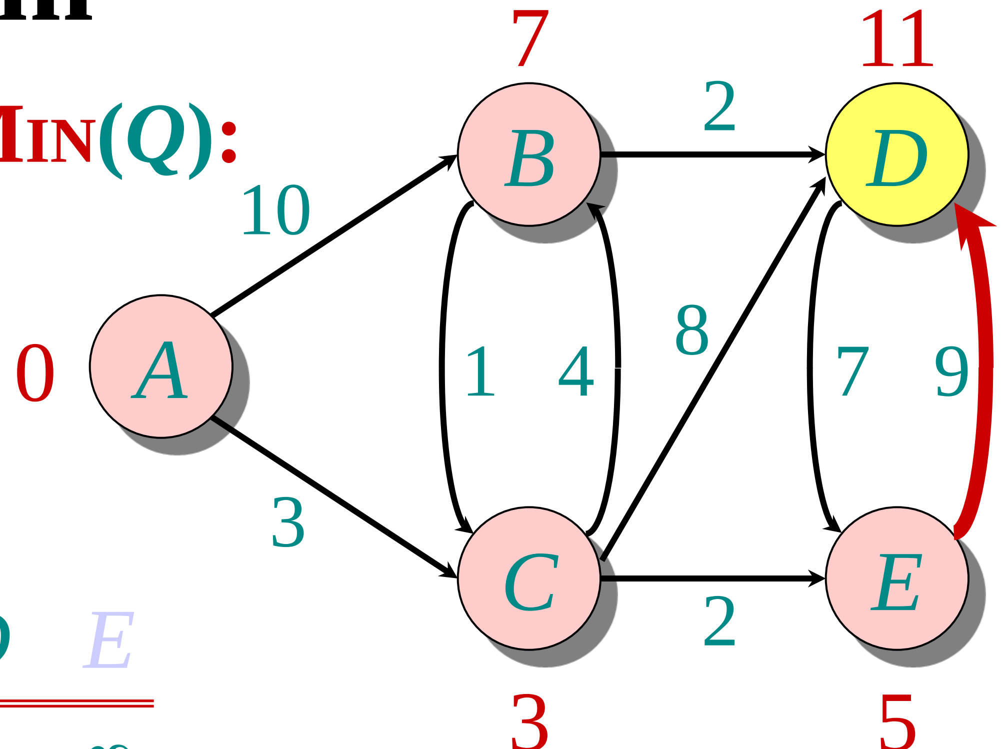
while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

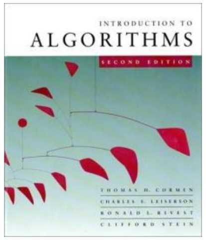
"*B*" ← EXTRACT-MIN(*Q*):

S: { *A*, *C*, *E*, *B* }



Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

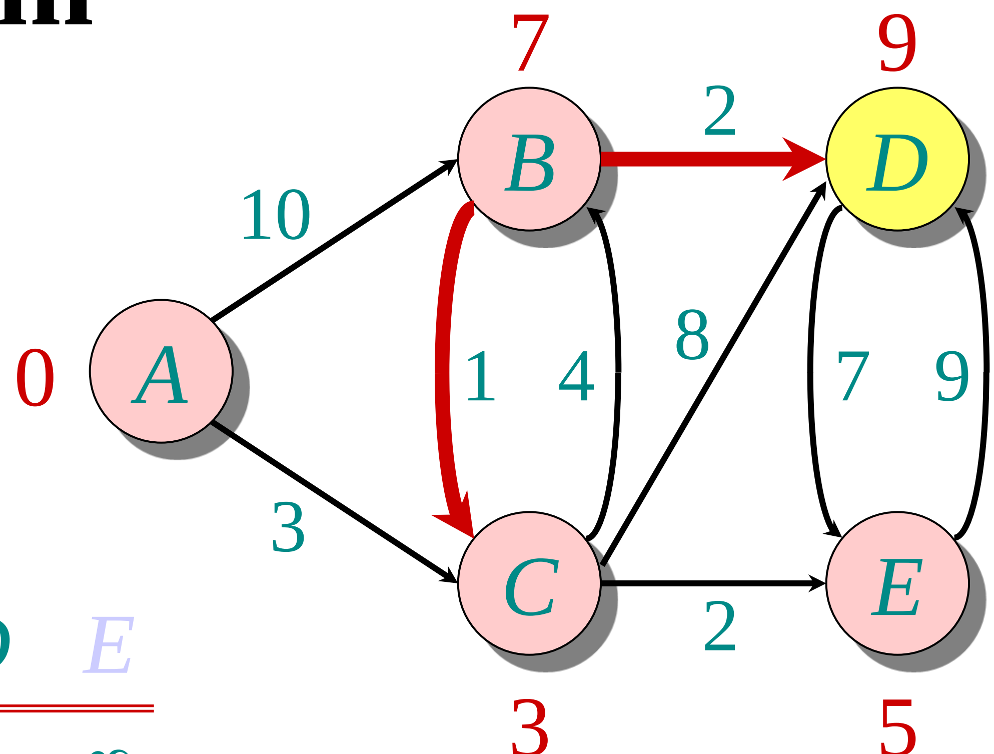
while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

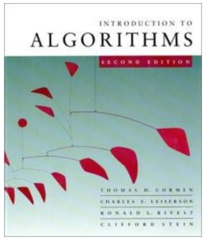
Relax all edges leaving *B*:

S: { *A*, *C*, *E*, *B* }



Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

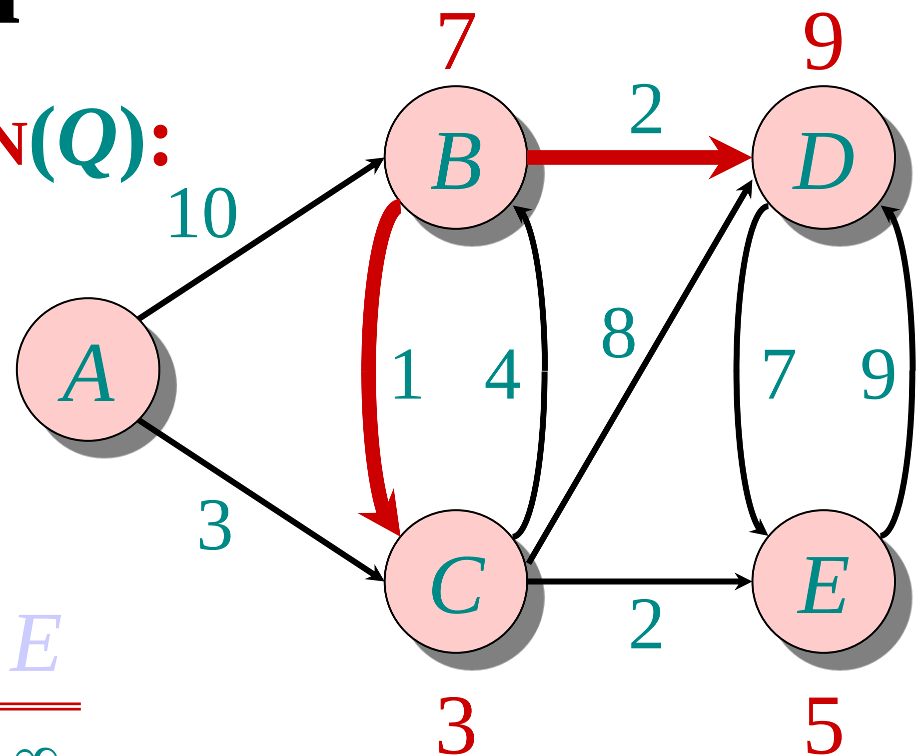
while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$



Example of Dijkstra's algorithm

"D" ← EXTRACT-MIN(Q):

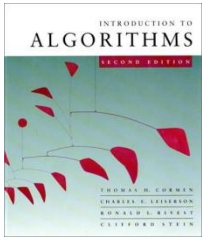
S: { A, C, E, B, D }



Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	
			9	

```
while Q ≠ ∅ do
    u ← EXTRACT-MIN(Q)
    S ← S ∪ {u}
    for each v ∈ Adj[u] do
        if d[v] > d[u] + w(u, v) then
            d[v] ← d[u] + w(u, v)
```



Analysis of Dijkstra

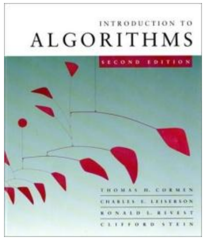
$|V|$ times:

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  $degree(u)$ times:
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$

Handshaking Lemma $\Rightarrow \Theta(|E|)$ implicit DECREASE-KEY's.

Time $= \Theta(|V|) \cdot T_{\text{EXTRACT-MIN}} + \Theta(|E|) \cdot T_{\text{DECREASE-KEY}}$

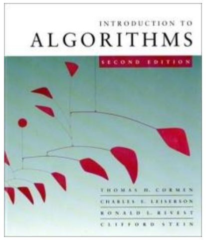
Note: Same formula as in the analysis of Prim's minimum spanning tree algorithm.



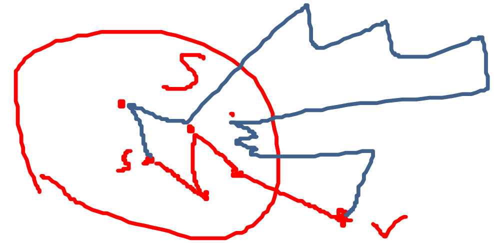
Analysis of Dijkstra (continued)

Time = $\Theta(|V|) \cdot T_{\text{EXTRACT-MIN}} + \Theta(|E|) \cdot T_{\text{DECREASE-KEY}}$

Q	$T_{\text{EXTRACT-MIN}}$	$T_{\text{DECREASE-KEY}}$	Total
array	$O(\|V\|)$	$O(1)$	$O(\|V\|^2)$
binary heap	$O(\log \|V\|)$	$O(\log \|V\|)$	$O(\|E\| \log \|V\|)$
Fibonacci heap	$O(\log \|V\|)$ amortized	$O(1)$ amortized	$O(\|E\| + \|V\| \log \|V\|)$ worst case

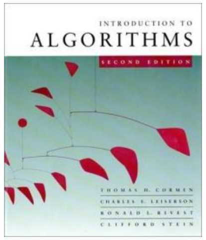


Correctness



Theorem. (i) For all $v \in S$: $d[v] = \delta(s, v)$
(ii) For all $v \notin S$: $d[v]$ = weight of shortest path from s to v that uses only (besides v itself) vertices in S.

Corollary. Dijkstra's algorithm terminates with $d[v] = \delta(s, v)$ for all $v \in V$.

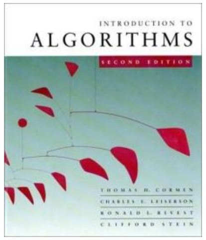


Correctness

Theorem. (i) For all $v \in S$: $d[v] = \delta(s, v)$
(ii) For all $v \notin S$: $d[v]$ = weight of shortest path from s to v that uses only (besides v itself) vertices in S.

Proof. By induction.

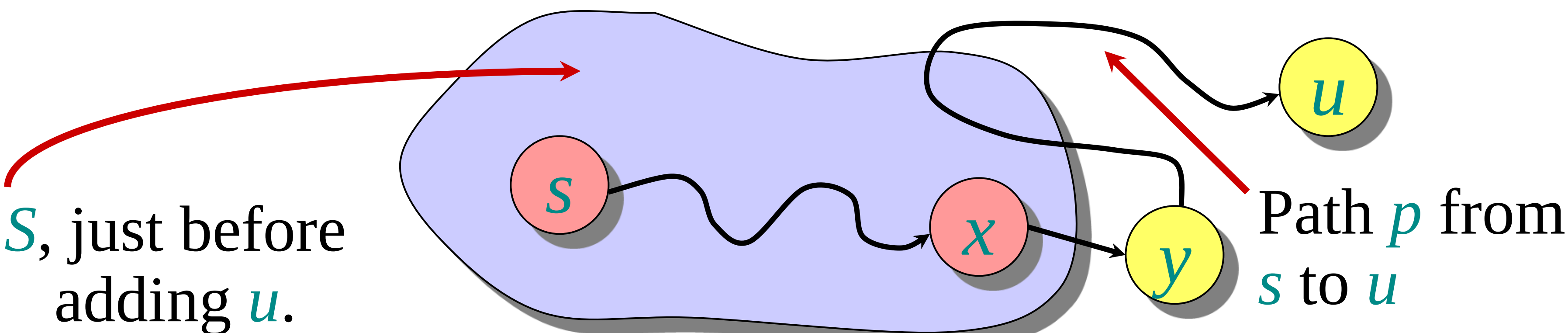
- Base: Before the while loop, $d[s]=0$ and $d[v]=\infty$ for all $v \neq s$, so (i) and (ii) are true.
- Step: Assume (i) and (ii) are true before an iteration; now we need to show they remain true after another iteration. Let u be the vertex added to S, so $d[u] \leq d[v]$ for all other $v \notin S$.

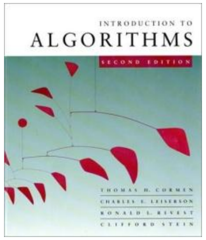


Correctness

Theorem. (i) For all $v \in S$: $d[v] = \delta(s, v)$
(ii) For all $v \notin S$: $d[v]$ = weight of shortest path from s to v that uses only (besides v itself) vertices in S.

- (i) Need to show that $d[u] = \delta(s, u)$. Assume the contrary.
$\Rightarrow$ There is a path p from s to u with $w(p) < d[u]$. Because of (ii) that path uses vertices $\notin S$, in addition to u.
$\Rightarrow$ Let y be first vertex on p such that $y \notin S$.





Correctness

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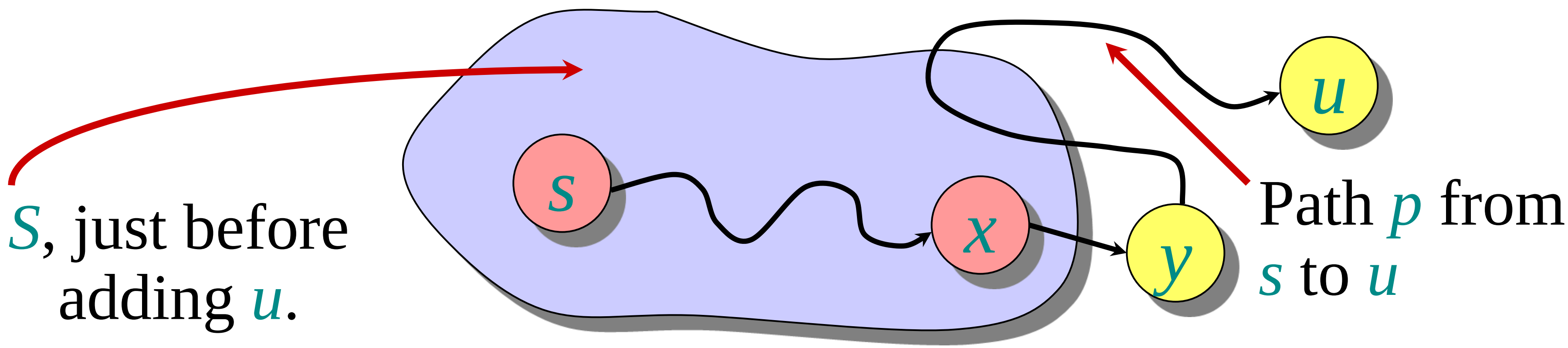
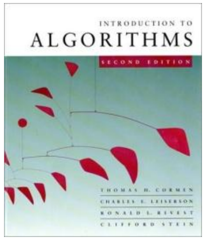


$\Rightarrow d[y] \leq w(p) < d[u]$. Contradiction to the choice of u.

weights are nonnegative

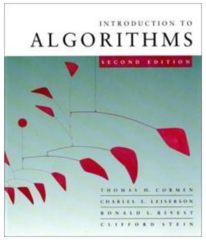
assumption about path



Correctness

Theorem. (i) For all $v \in S$: $d[v] = \delta(s, v)$
(ii) For all $v \notin S$: $d[v]$ = weight of shortest path from s to v that uses only (besides v itself) vertices in S.

- (ii) Let $v \notin S$. Let p be a shortest path from s to v that uses only (besides v itself) vertices in S.
    - p does not contain u: (ii) true by inductive hypothesis
    - p contains u: p consists of vertices in $S \setminus \{u\}$ and ends with an edge from u to v.
      $\Rightarrow$ $w(p)=d[u]+w(u,v)$, which is the value of $d[v]$ after adding u. So (ii) is true.



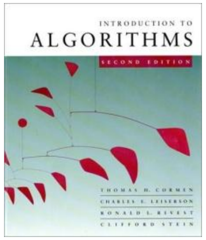
Unweighted graphs

Suppose $w(u, v) = 1$ for all $(u, v) \in E$. Can the code for Dijkstra be improved?

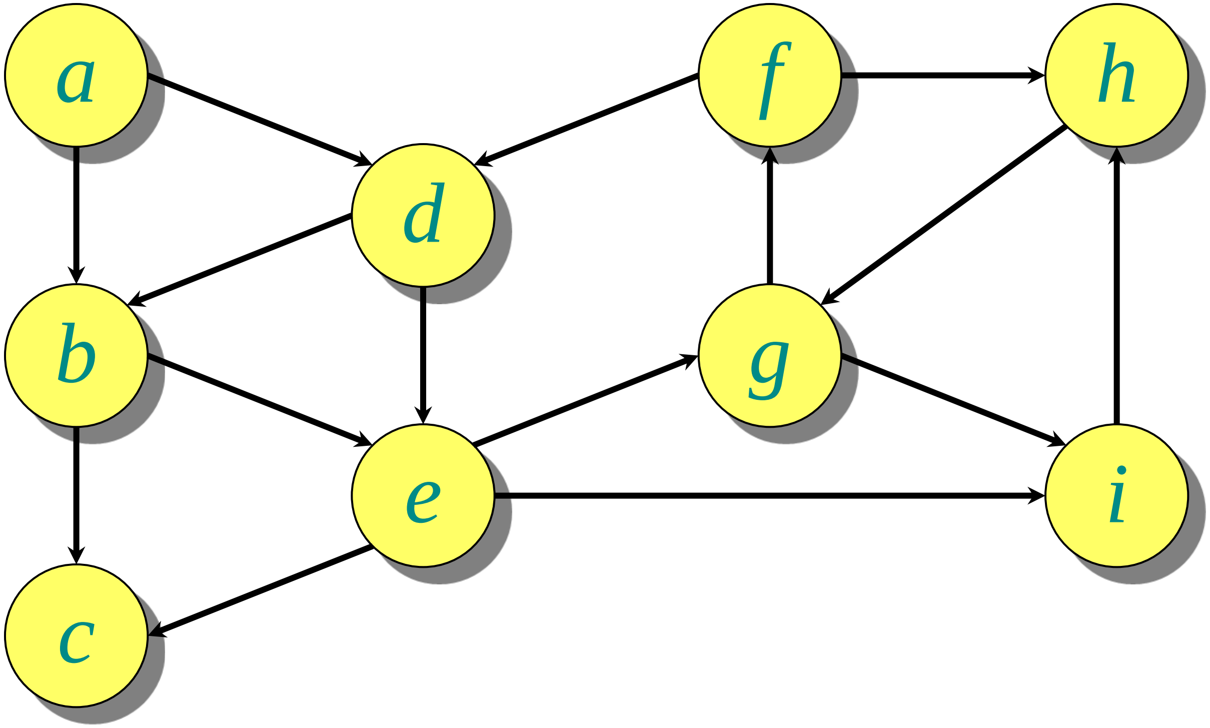
- Use a simple FIFO queue instead of a priority queue.
- ***Breadth-first search***

while $Q \neq \emptyset$
    do $u \leftarrow$ DEQUEUE(Q)
        for each $v \in Adj[u]$
            do if $d[v] = \infty$
                then $d[v] \leftarrow d[u] + 1$
                    ENQUEUE(Q, v)

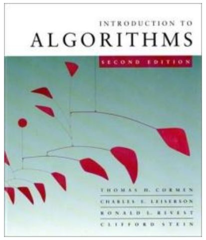
Analysis: Time $= O(|V| + |E|)$.



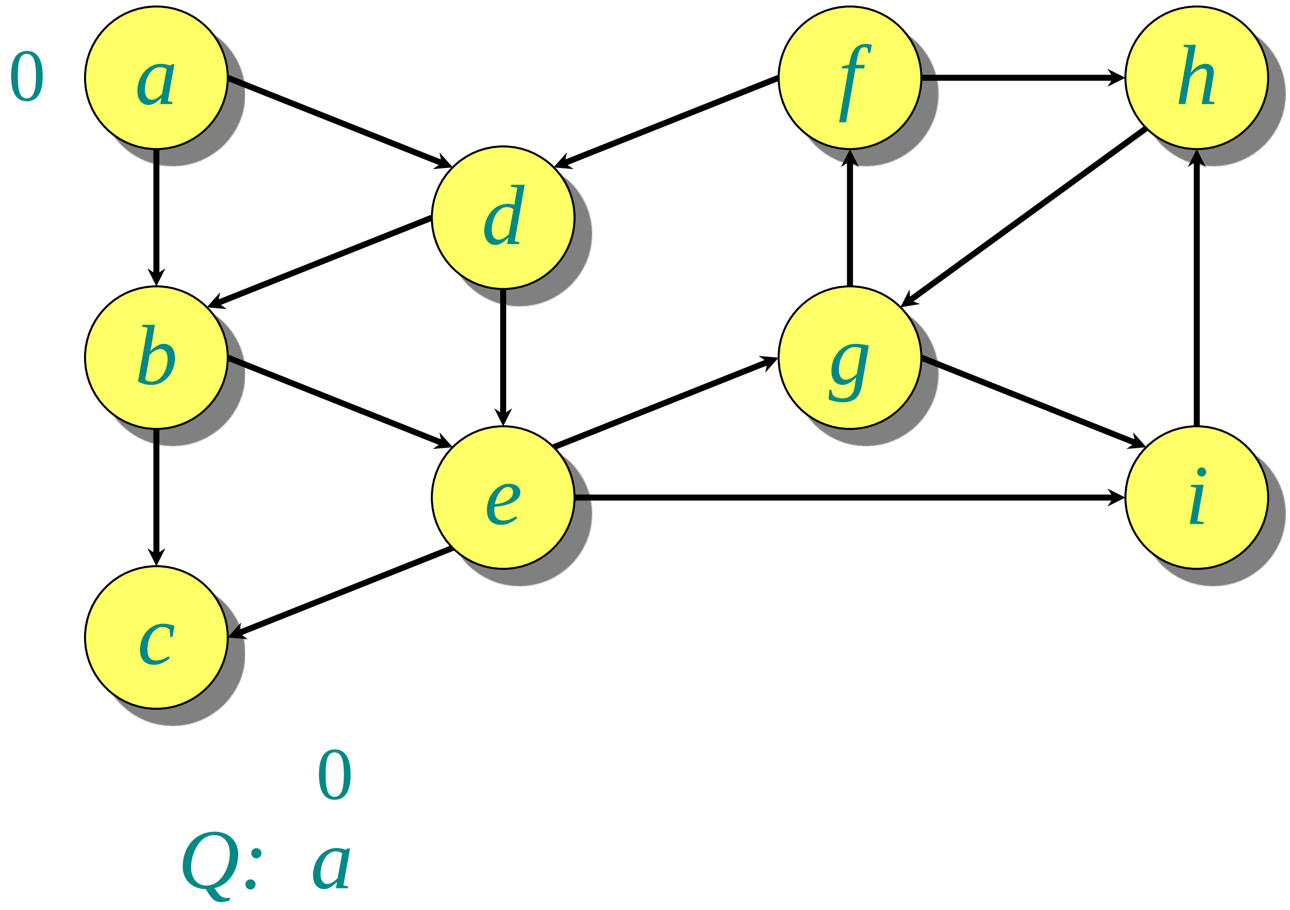
Example of breadth-first search

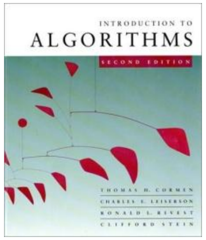


Q:

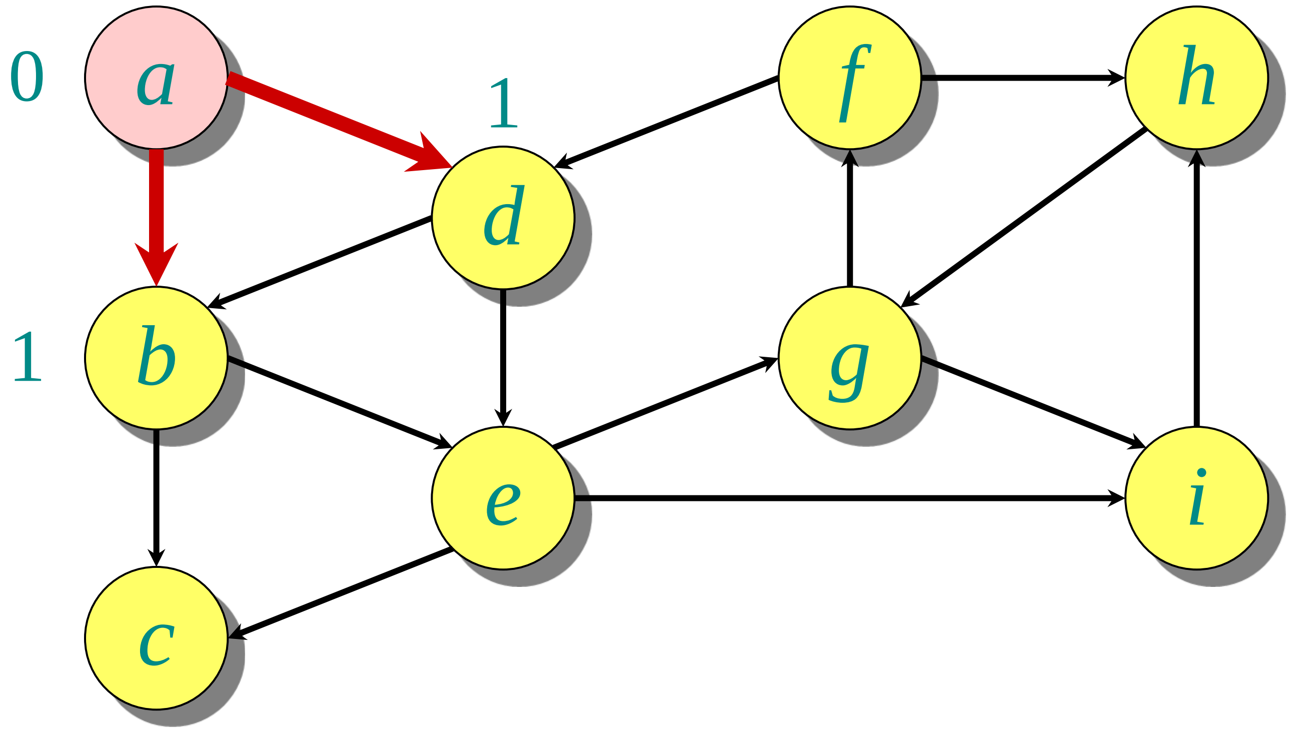


Example of breadth-first search



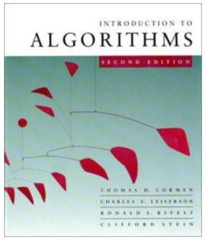


Example of breadth-first search

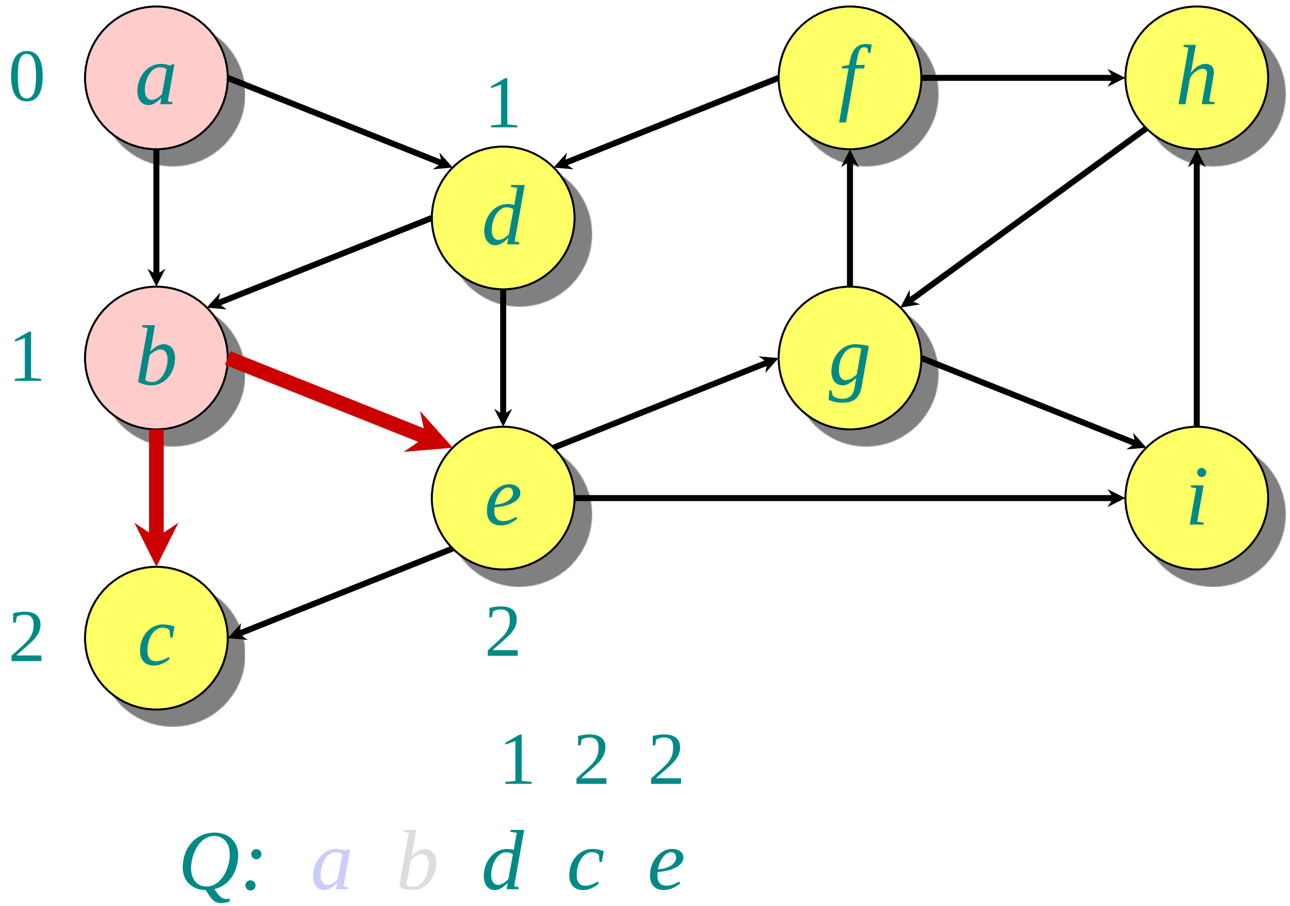


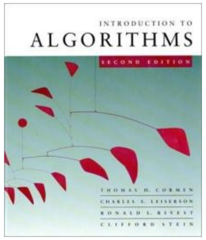
1 1

Q: a b d

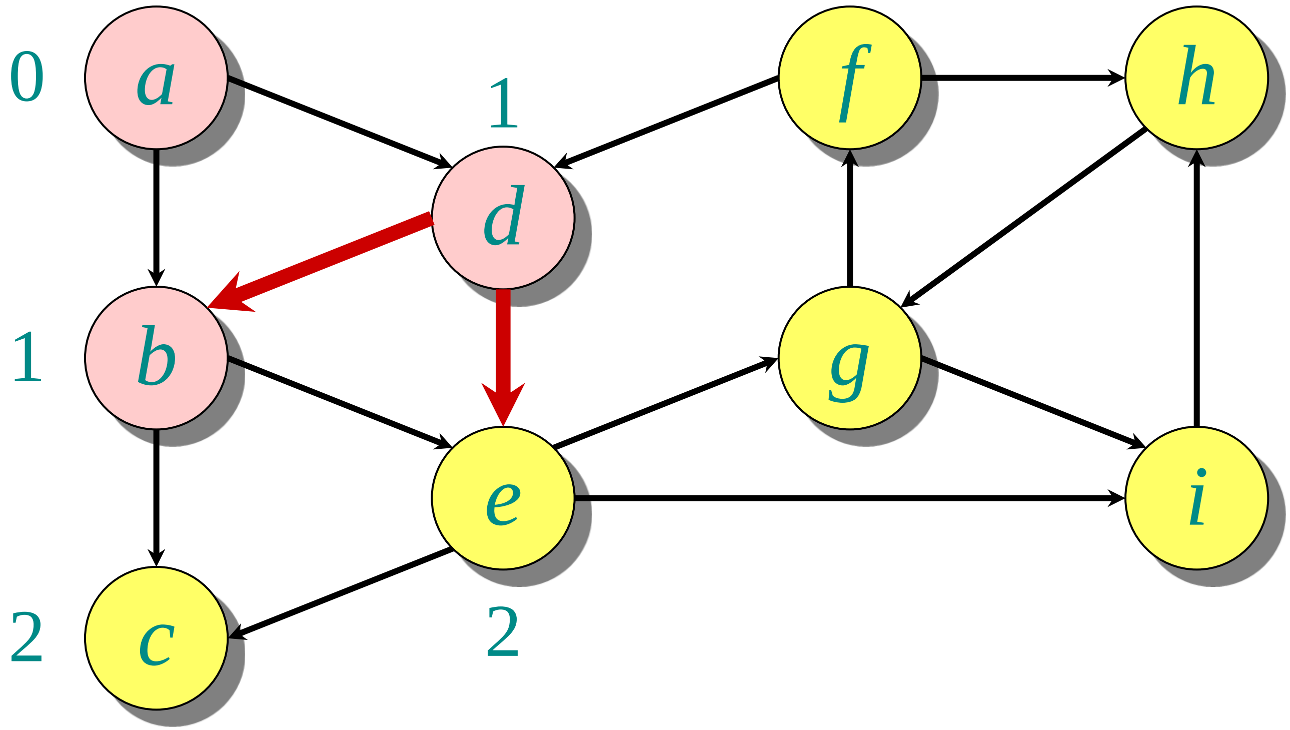


Example of breadth-first search



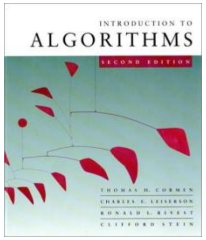


Example of breadth-first search

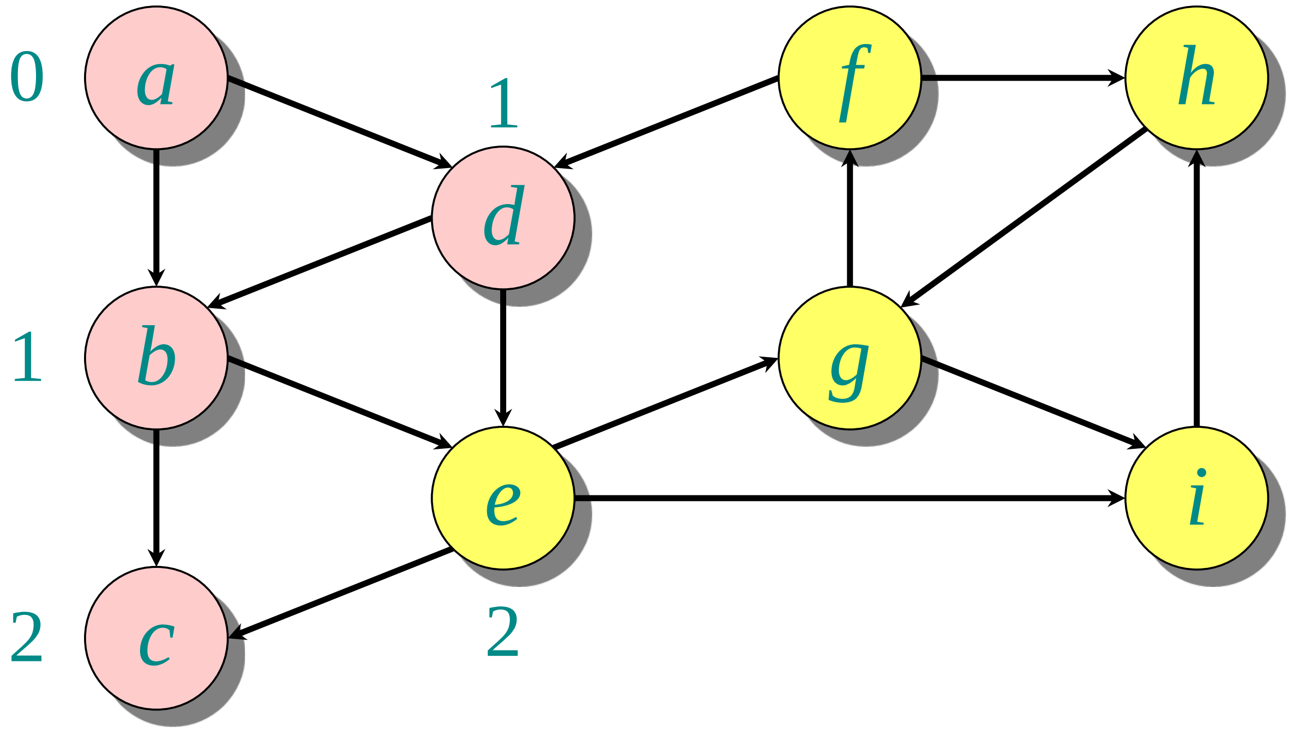


2 2

Q: *a b d c e*

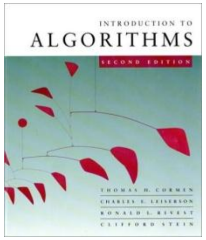


Example of breadth-first search

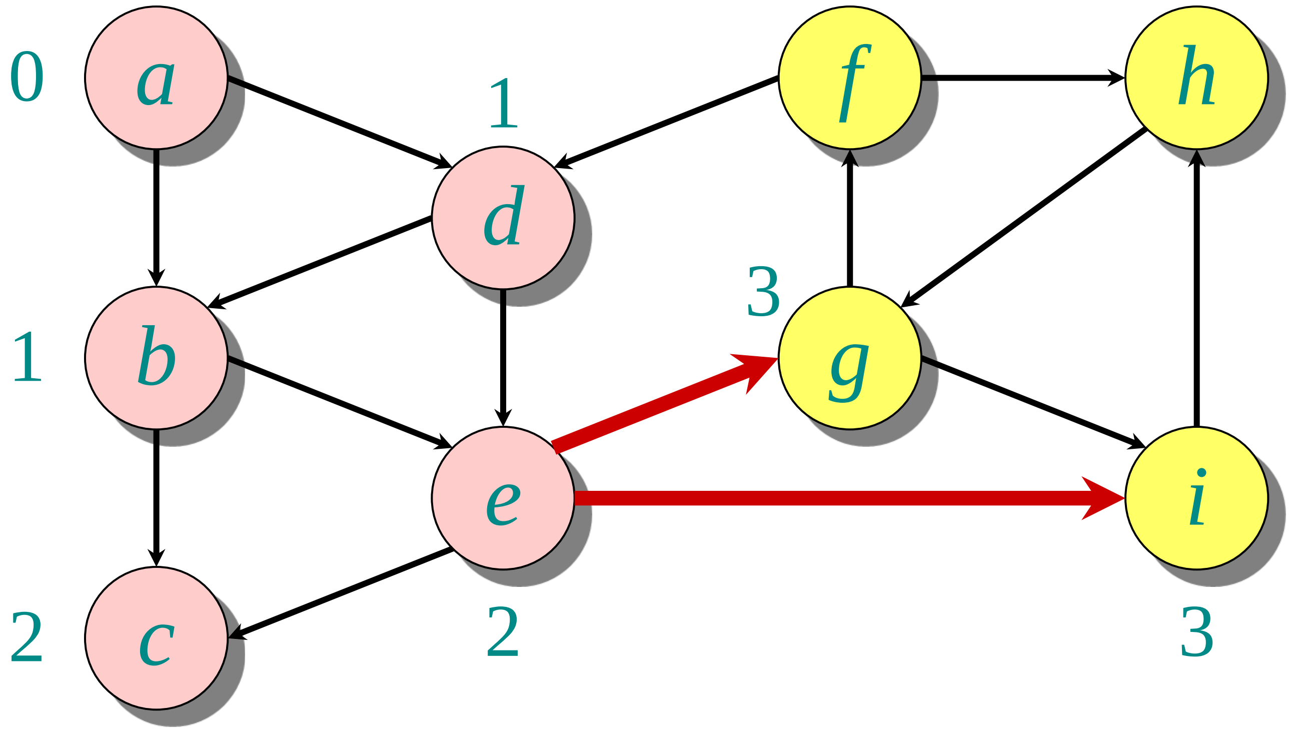


2

Q: *a* *b* *d* *c* *e*

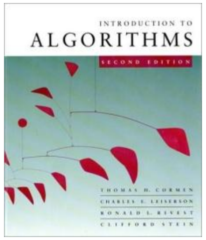


Example of breadth-first search

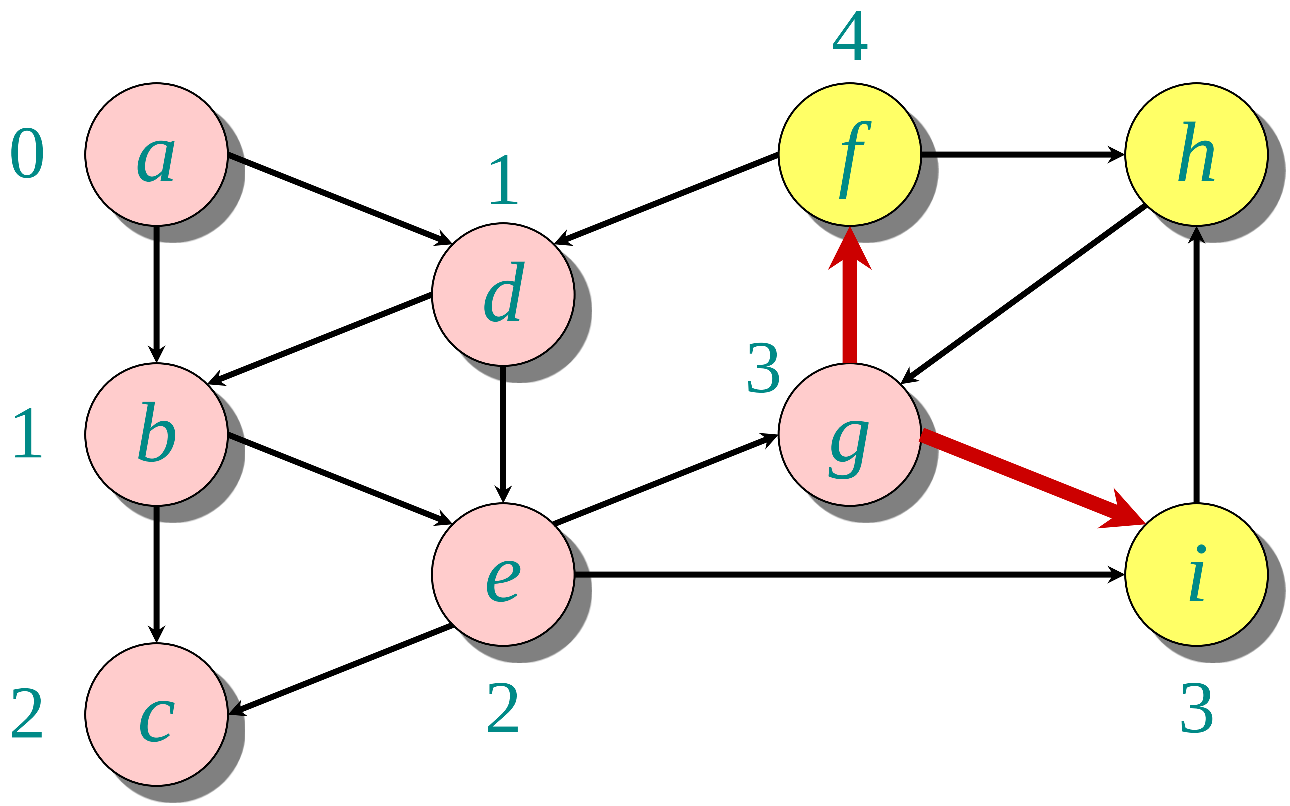


3 3

Q: a b d c e g i

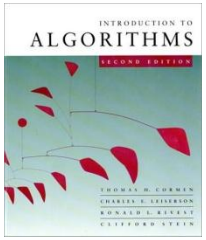


Example of breadth-first search

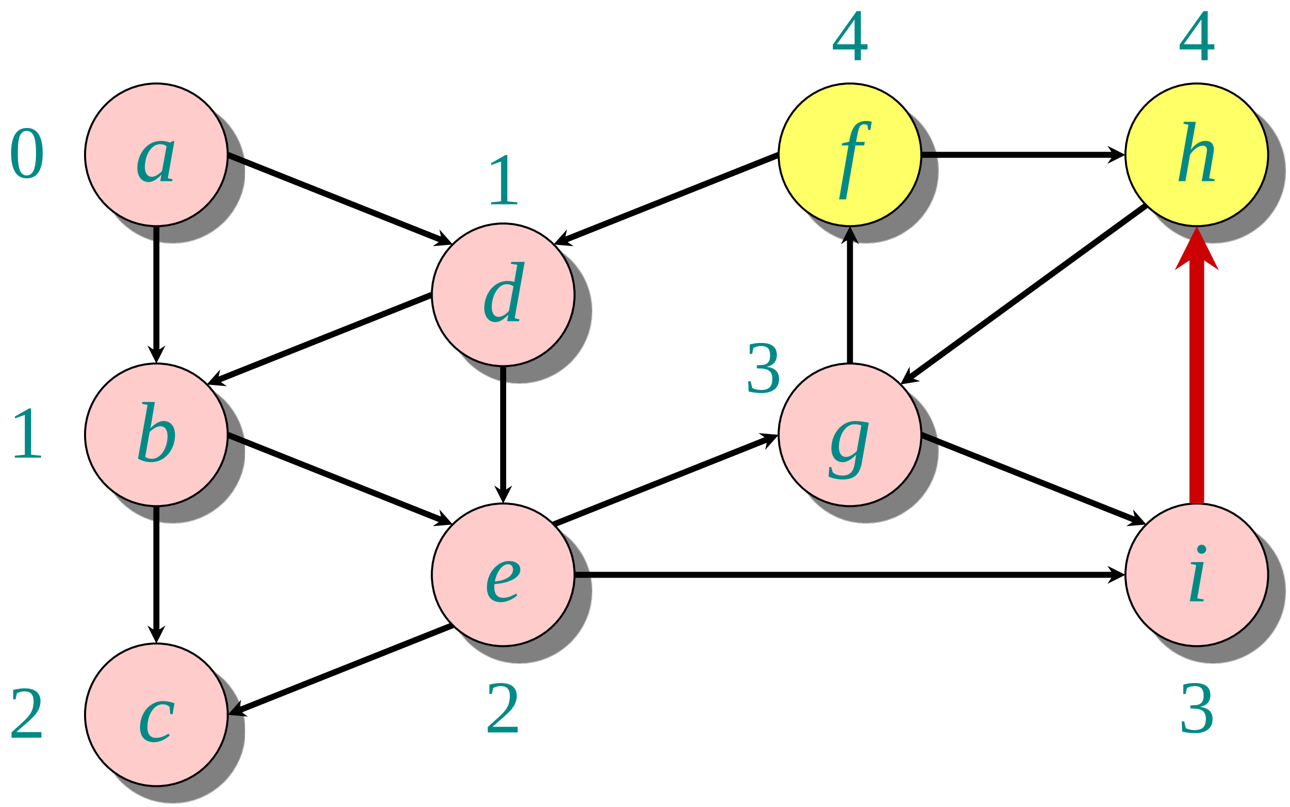


3 4

Q: *a b d c e g i f*

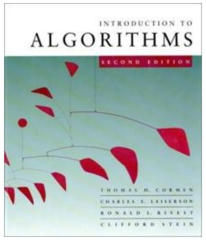


Example of breadth-first search

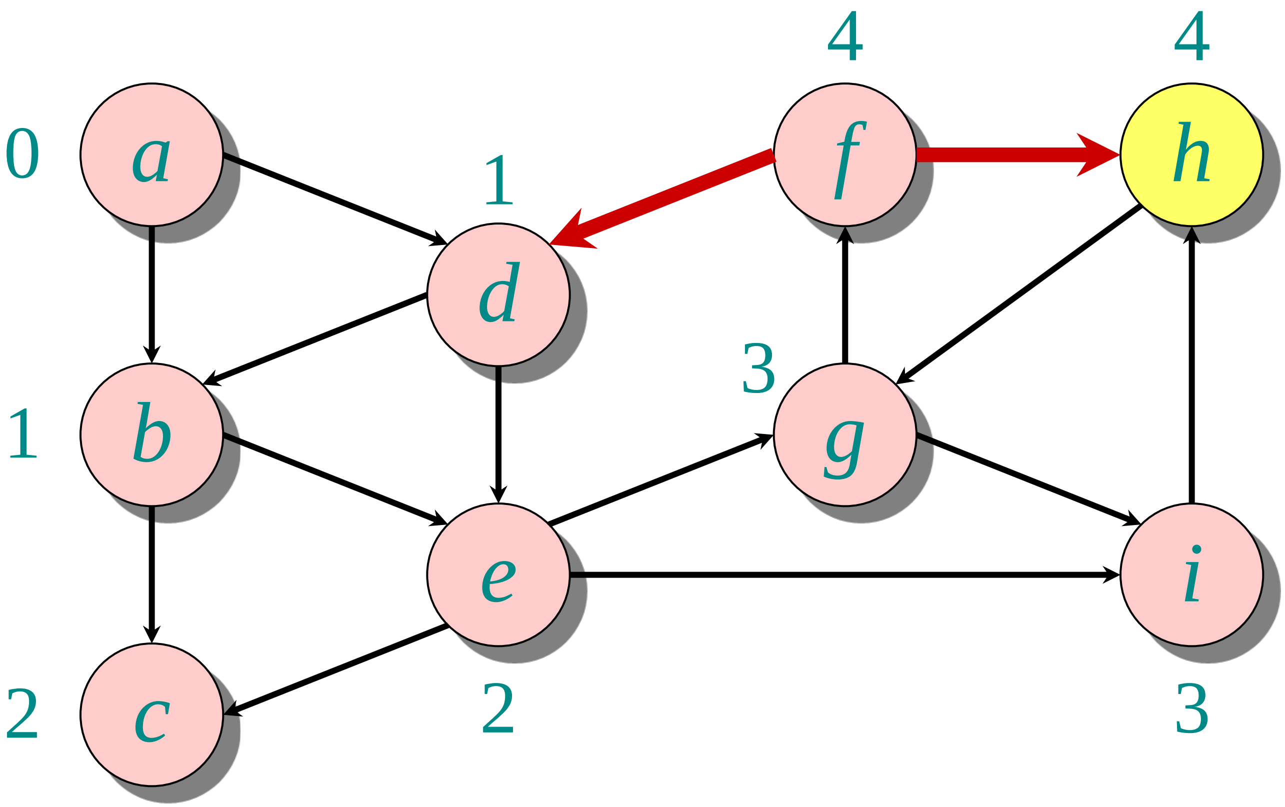


4 4

Q: *a b d c e g i f h*

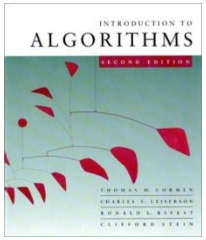


Example of breadth-first search

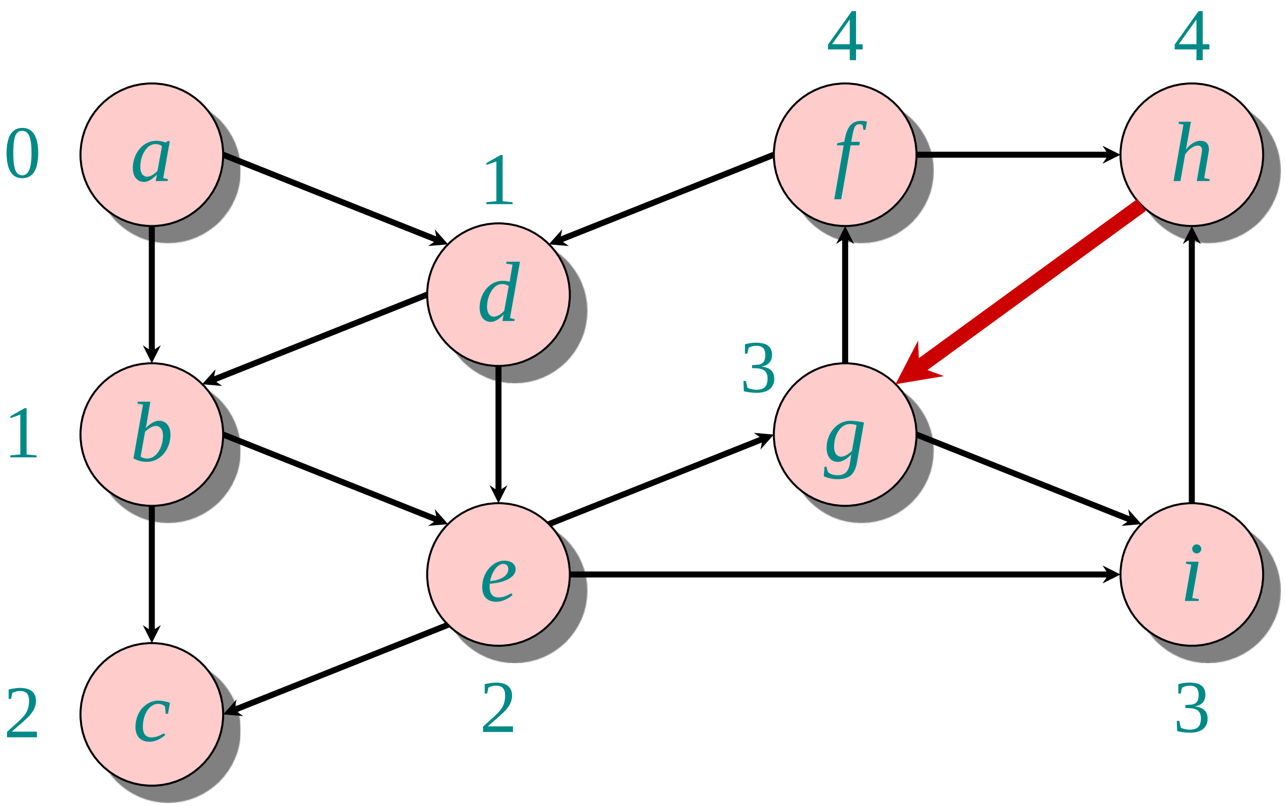


4

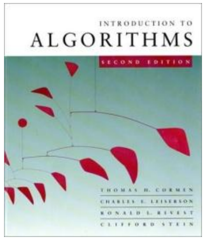
Q: *a b d c e g i f h*



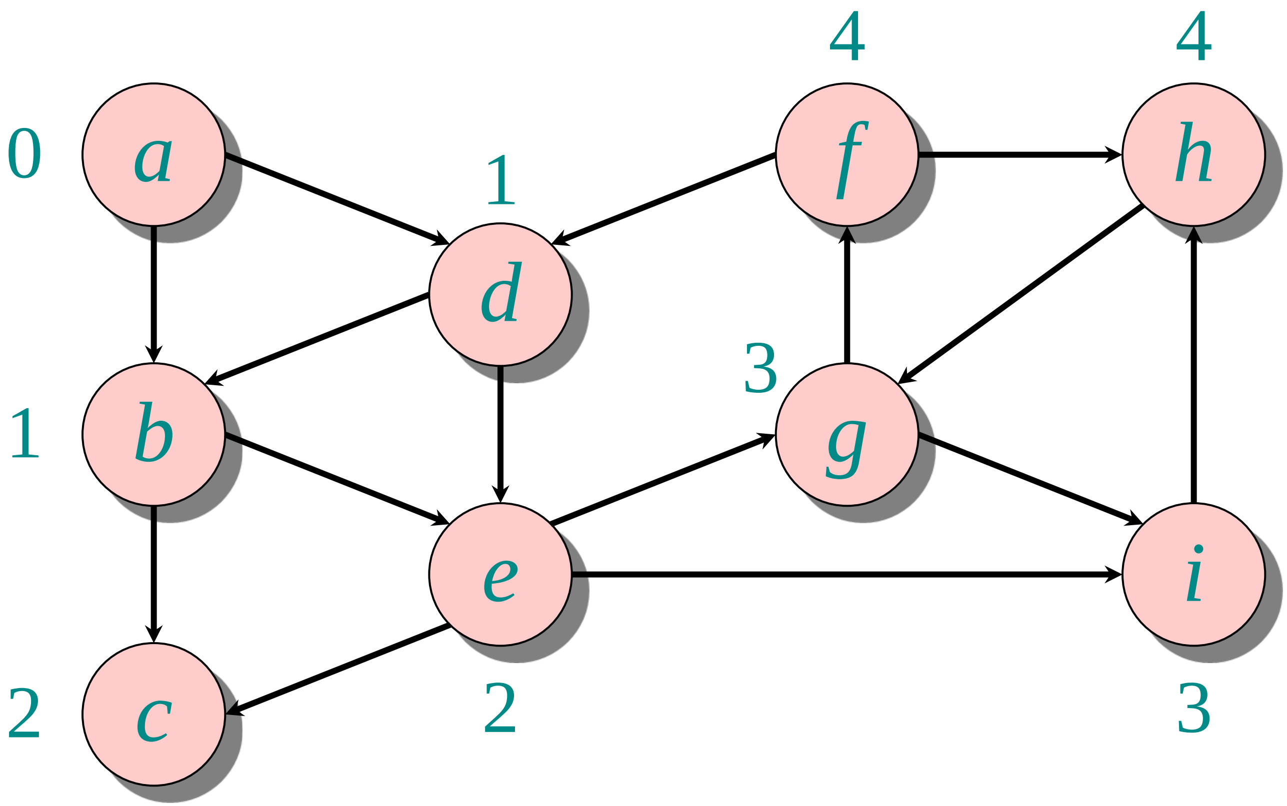
Example of breadth-first search



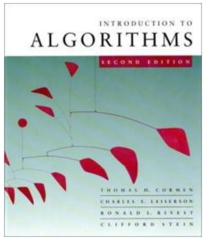
Q: *a b d c e g i f h*



Example of breadth-first search



Q: *a b d c e g i f h*



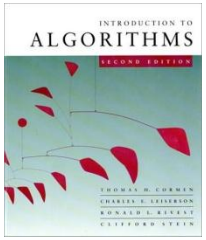
Correctness of BFS

while $Q \neq \emptyset$
    do $u \leftarrow$ DEQUEUE(Q)
        for each $v \in Adj[u]$
            do if $d[v] = \infty$
                then $d[v] \leftarrow d[u] + 1$
                    ENQUEUE(Q, v)

Key idea:

The FIFO Q in breadth-first search mimics the priority queue Q in Dijkstra.

- **Invariant:** v comes after u in Q implies that $d[v] = d[u]$ or $d[v] = d[u] + 1$.



How to find the actual shortest paths?

Store a predecessor tree:

$d[s] \leftarrow 0$

for each $v \in V - \{s\}$

  do $d[v] \leftarrow \infty$

$S \leftarrow \emptyset$

$Q \leftarrow V$ ⊳ Q is a priority queue maintaining $V - S$

while $Q \neq \emptyset$

  do $u \leftarrow$ EXTRACT-MIN(Q)

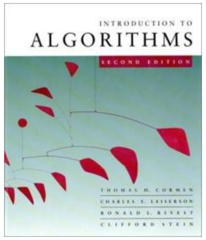
    $S \leftarrow S \cup \{u\}$

    for each $v \in Adj[u]$

      do if $d[v] > d[u] + w(u, v)$

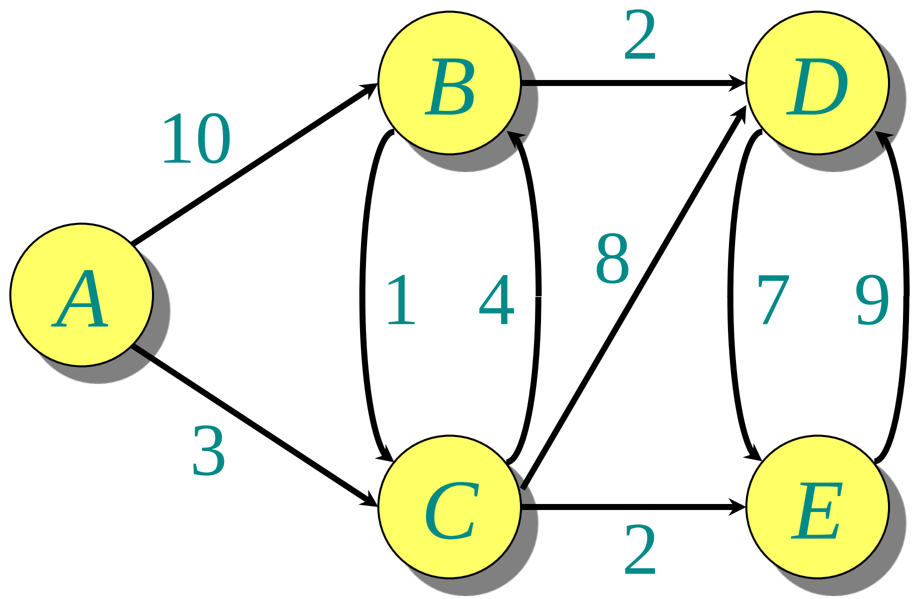
        then $d[v] \leftarrow d[u] + w(u, v)$

          $\pi[v] \leftarrow u$

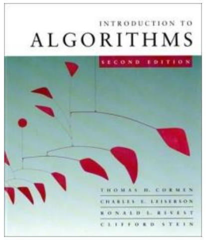


Example of Dijkstra's algorithm

Graph with nonnegative edge weights:



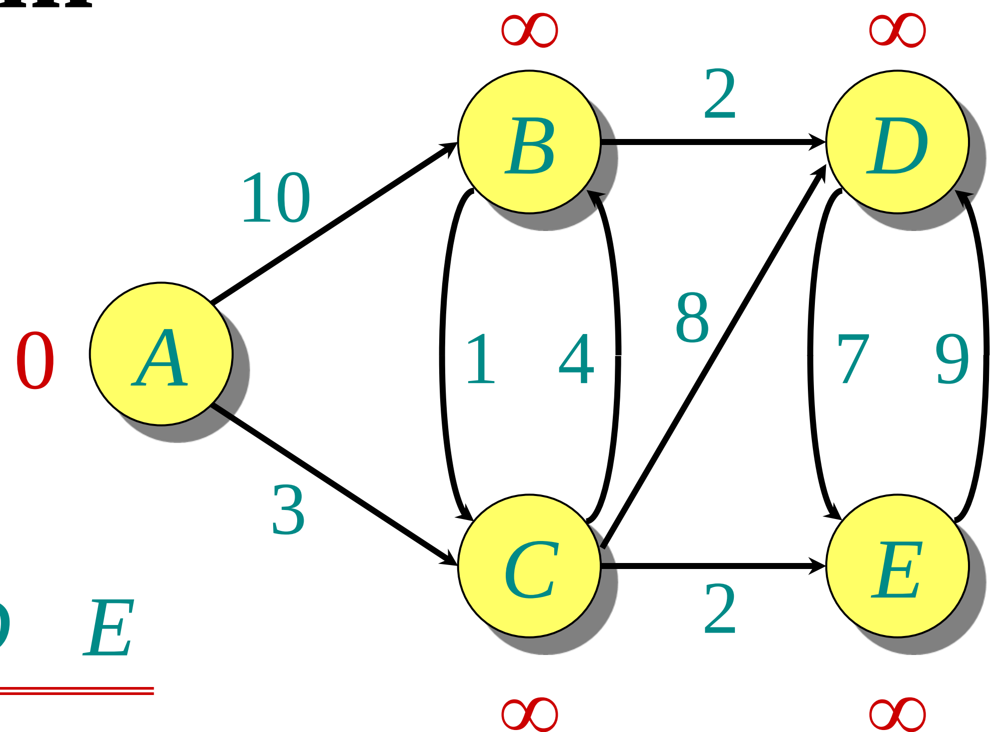
while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



Example of Dijkstra's algorithm

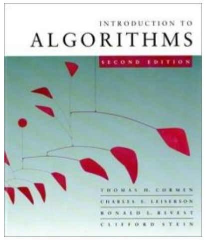
Initialize:

S: { }



Q:	A	B	C	D	E
	0	∞	∞	∞	∞

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



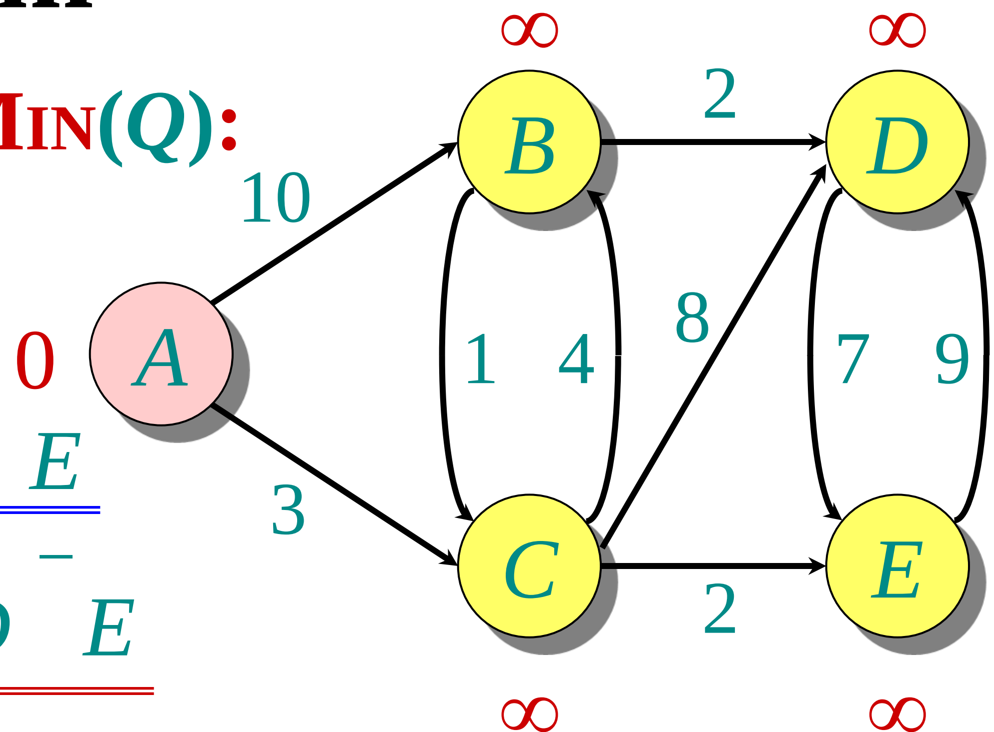
Example of Dijkstra's algorithm

"A" ← EXTRACT-MIN(Q):

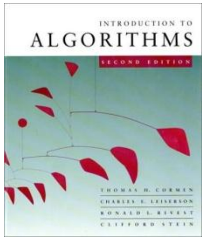
S: { A }

π:	A	B	C	D	E
	–	–	–	–	–

Q:	A	B	C	D	E
	0	∞	∞	∞	∞



while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



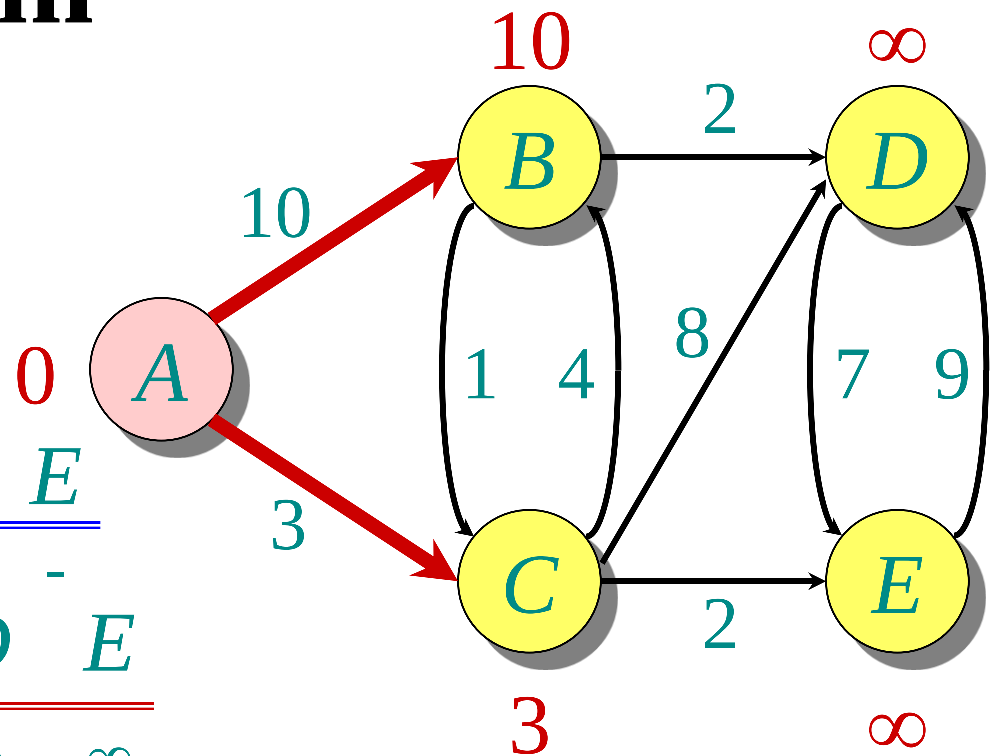
Example of Dijkstra's algorithm

Relax all edges leaving *A*:

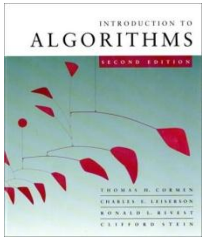
S: { *A* }

π:	A	B	C	D	E
	–	–	–	–	-

Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	–	–



while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



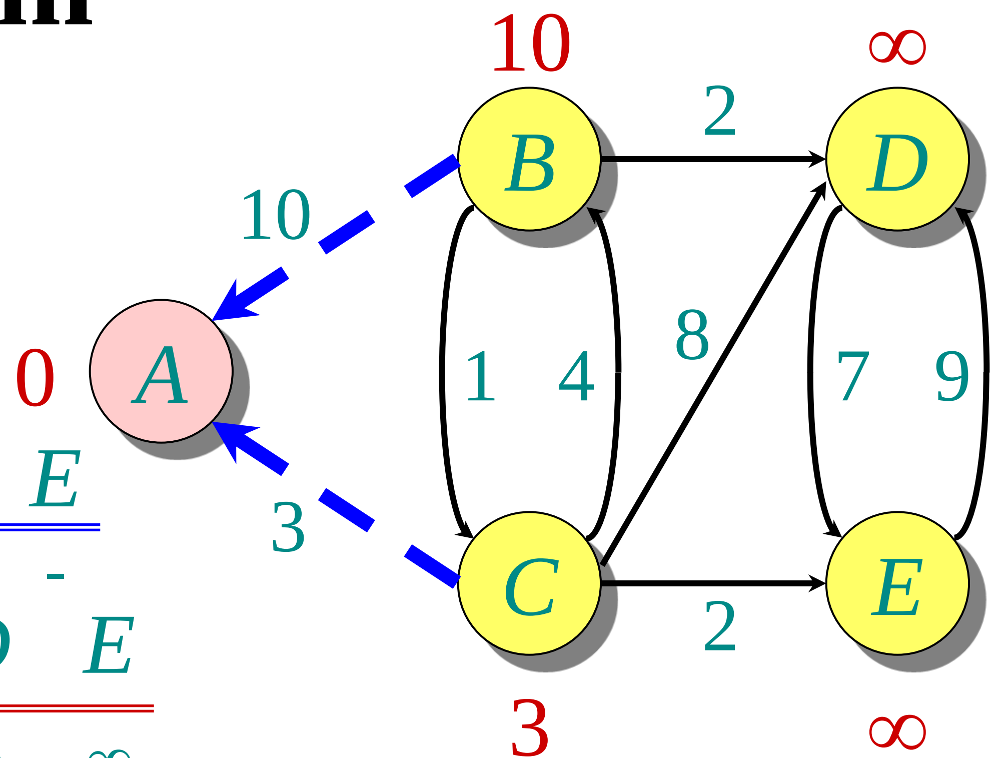
Example of Dijkstra's algorithm

Relax all edges leaving *A*:

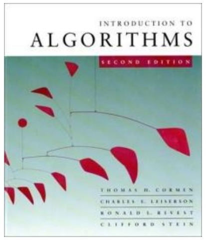
S: { *A* }

π:	*A*	*B*	*C*	*D*	*E*
	–	A	A	–	-

Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	–	–

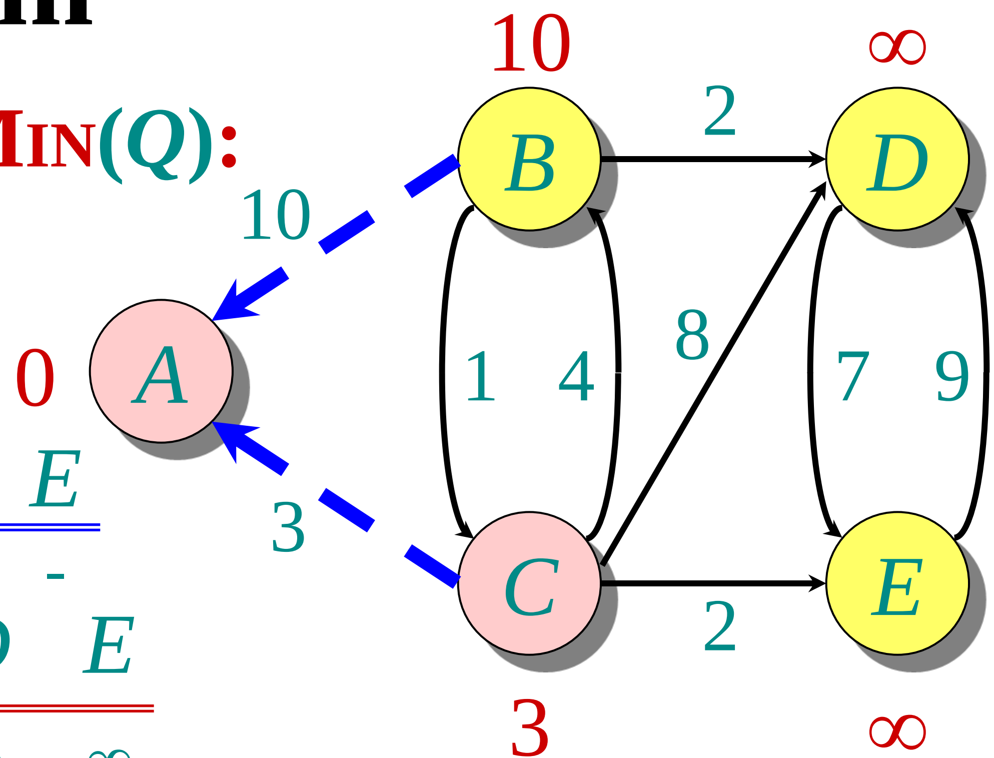


while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



Example of Dijkstra's algorithm

"*C*" ← EXTRACT-MIN(*Q*):



S: { *A*, *C* }

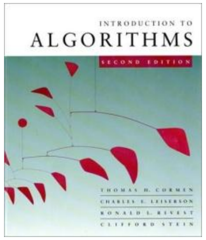
π:

A	*B*	*C*	*D*	*E*
–	A	A	–	-

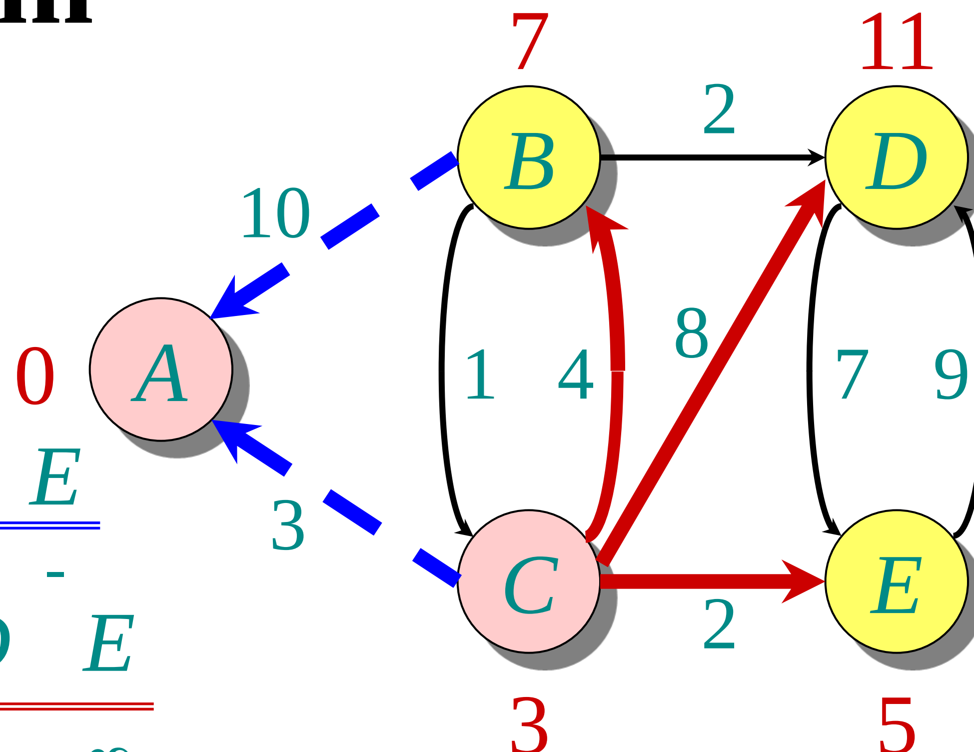
Q:

A	*B*	*C*	*D*	*E*
0	∞	∞	∞	∞
	10	3	–	–

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



Example of Dijkstra's algorithm



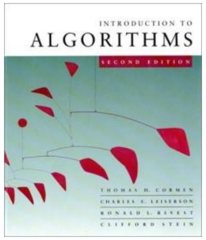
Relax all edges leaving *C*:

S: { *A*, *C* }

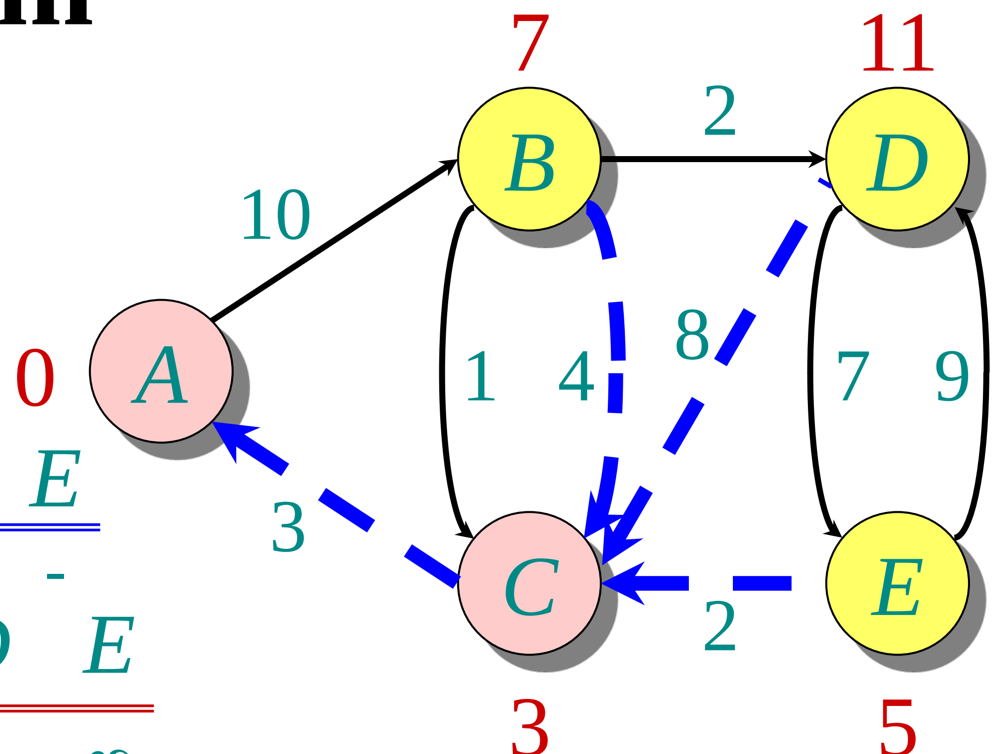
π:	A	B	C	D	E
	–	A	A	–	-

Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	–	–
		7		11	5

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



Example of Dijkstra's algorithm



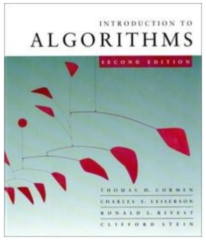
Relax all edges leaving *C*:

S: { *A*, *C* }

π:	*A*	*B*	*C*	*D*	*E*
	–	A	A	–	-

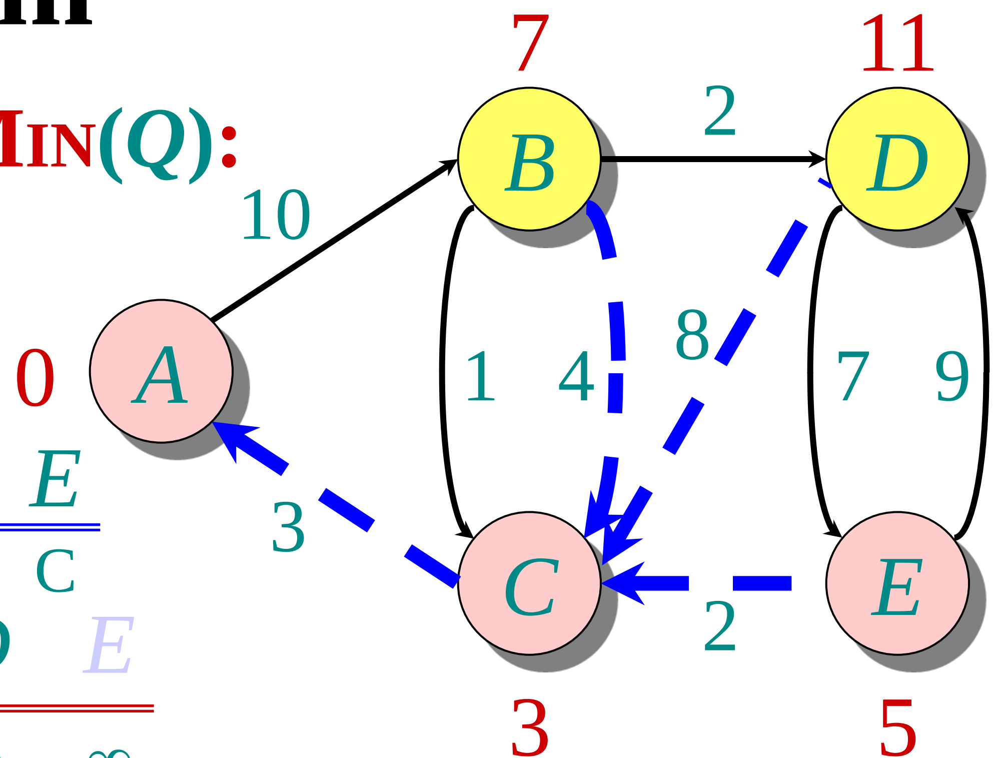
Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	–	–
		7		11	5

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



Example of Dijkstra's algorithm

"E" ← EXTRACT-MIN(Q):

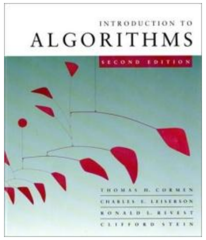


S: { A, C, E }

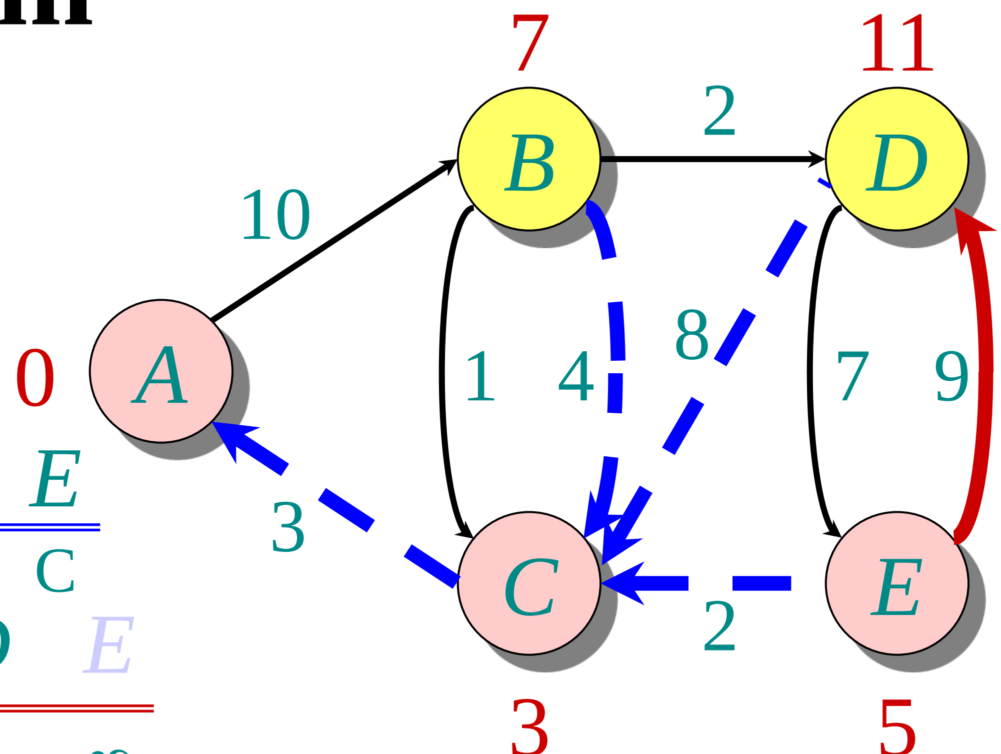
π:	A	B	C	D	E
	–	C	A	C	C

Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	–	–
		7		11	5

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



Example of Dijkstra's algorithm



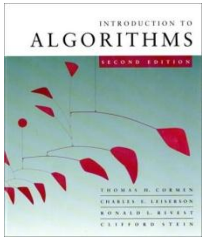
Relax all edges leaving *E*:

S: { A, C, E }

π:	A	B	C	D	E
	–	C	A	C	C

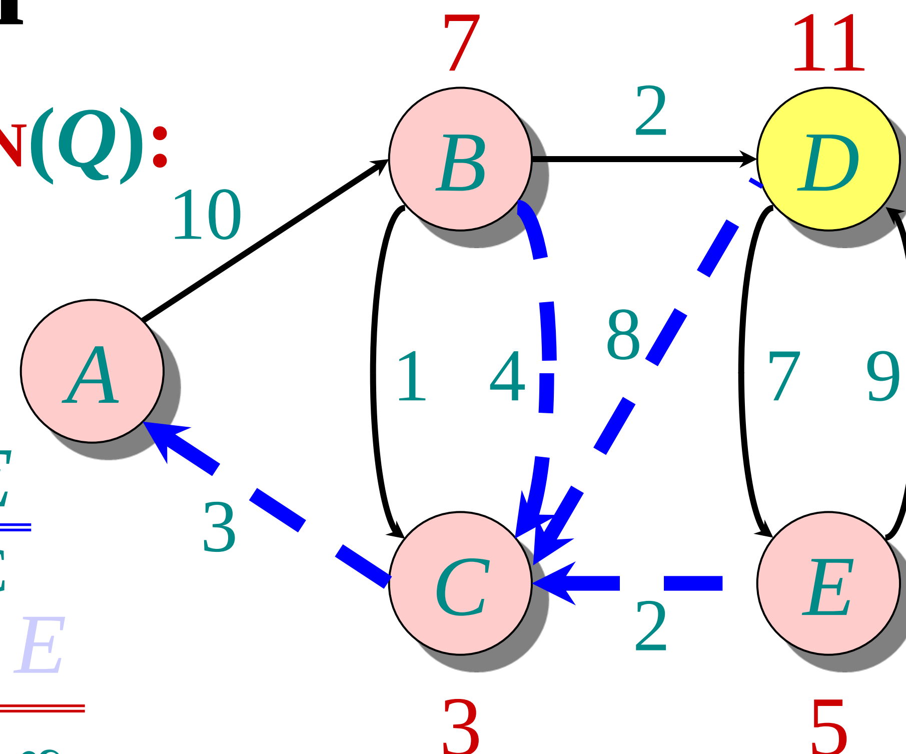
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



Example of Dijkstra's algorithm

"*B*" ← EXTRACT-MIN(*Q*):

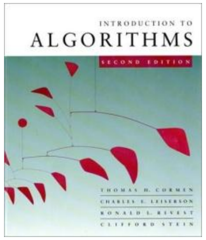


S: { *A*, *C*, *E*, *B* }

π:	*A*	*B*	*C*	*D*	*E*
	–	C	A	C	C

Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$



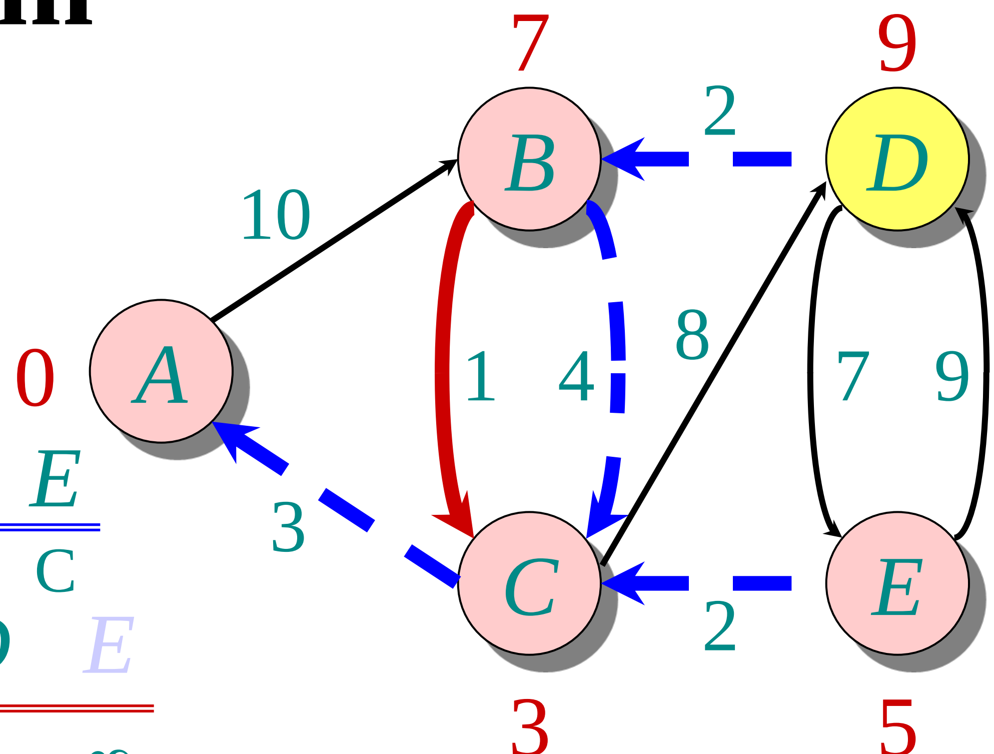
Example of Dijkstra’s algorithm

Relax all edges leaving *B*:

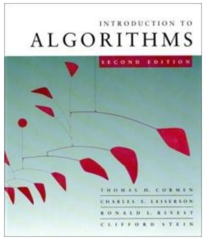
S: { *A*, *C*, *E*, *B* }

π:	*A*	*B*	*C*	*D*	*E*
	–	C	A	B	C

Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

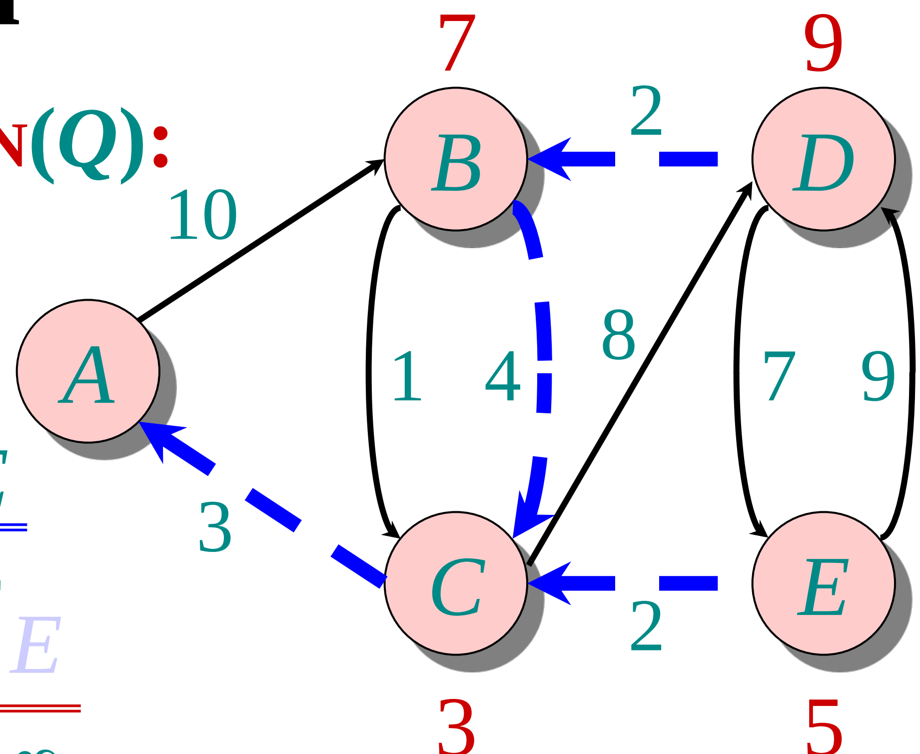



```
while Q ≠ ∅ do
    u ← EXTRACT-MIN(Q)
    S ← S ∪ {u}
    for each v ∈ Adj[u] do
        if d[v] > d[u] + w(u, v) then
            d[v] ← d[u] + w(u, v)
            π[v] ← u
```



Example of Dijkstra's algorithm

"*D*" ← EXTRACT-MIN(*Q*):



S: { *A*, *C*, *E*, *B*, *D* }

π:

A	*B*	*C*	*D*	*E*
–	C	A	B	C

Q:

A	*B*	*C*	*D*	*E*
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	
			9	

while $Q \neq \emptyset$ **do**
  $u \leftarrow$ EXTRACT-MIN(Q)
  $S \leftarrow S \cup \{u\}$
  for each $v \in Adj[u]$ **do**
    if $d[v] > d[u] + w(u, v)$ **then**
      $d[v] \leftarrow d[u] + w(u, v)$
      $\pi[v] \leftarrow u$