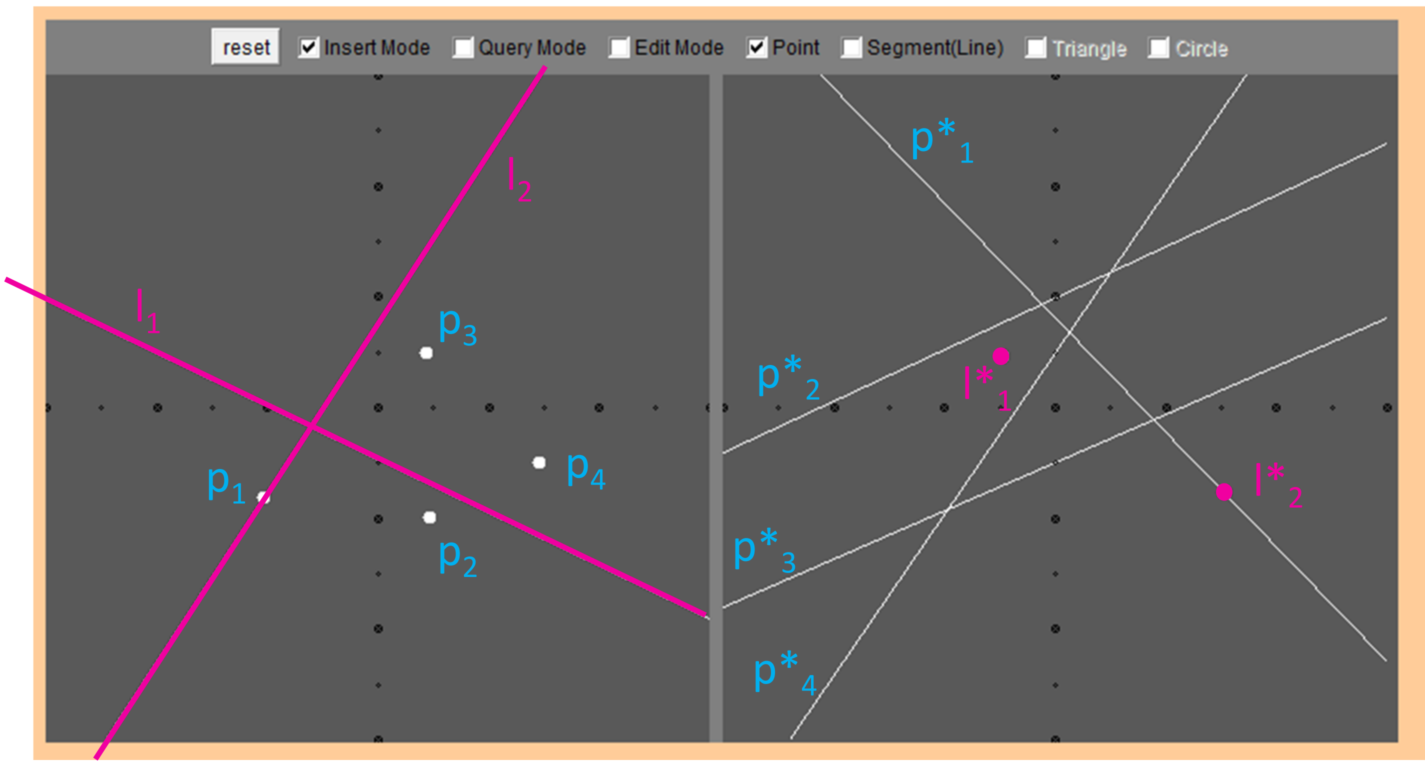
Point-Line Duality

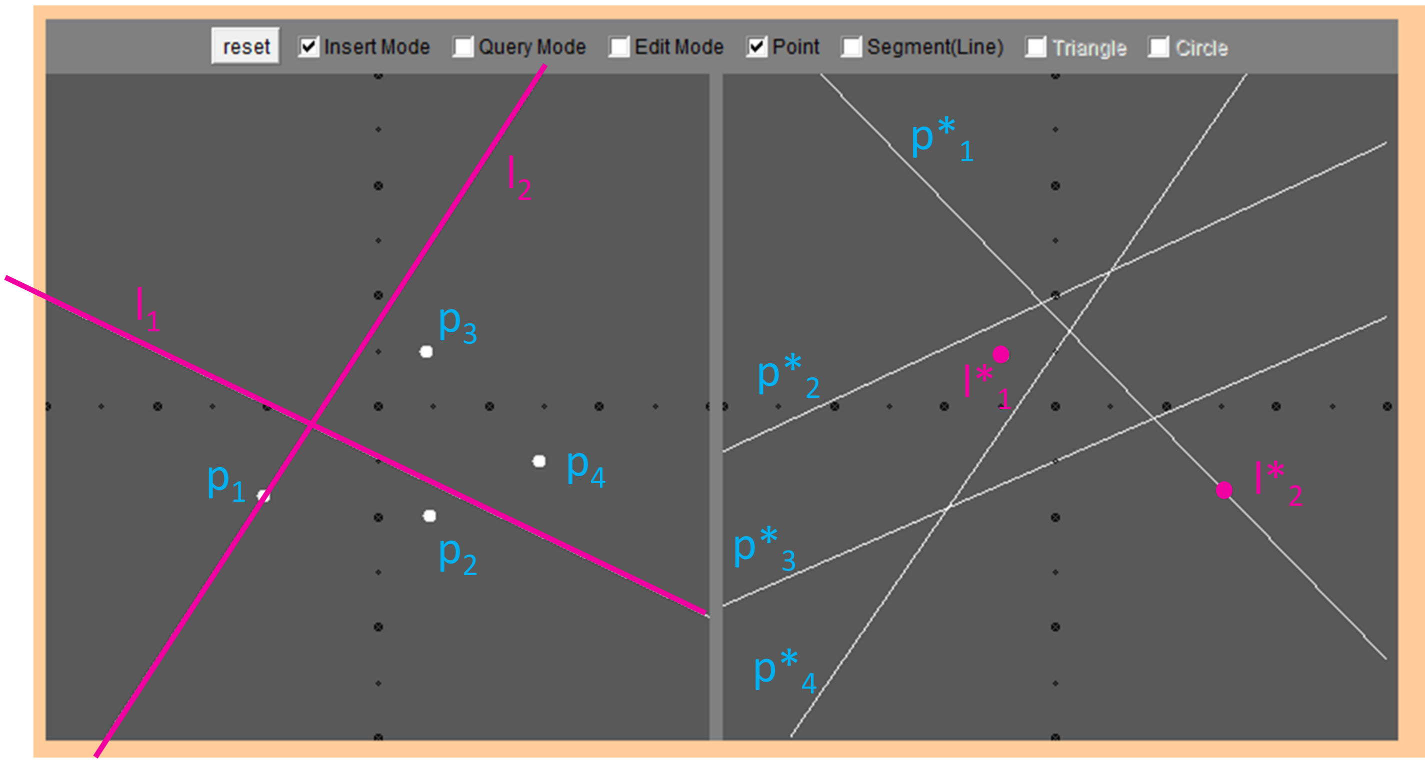
Let $P = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}$ be a set of n points. Now define a set $P^* = \{p_1^*, \ldots, p_n^*\}$ of n lines as follows:

Primal plane
Point: $p = (p_x, p_y)$
Line: $l: y = mx + b$

Dual plane
Line: $p^*: y = p_x x \ - p_y$
Point: $l^* = (m, -b)$



Properties:



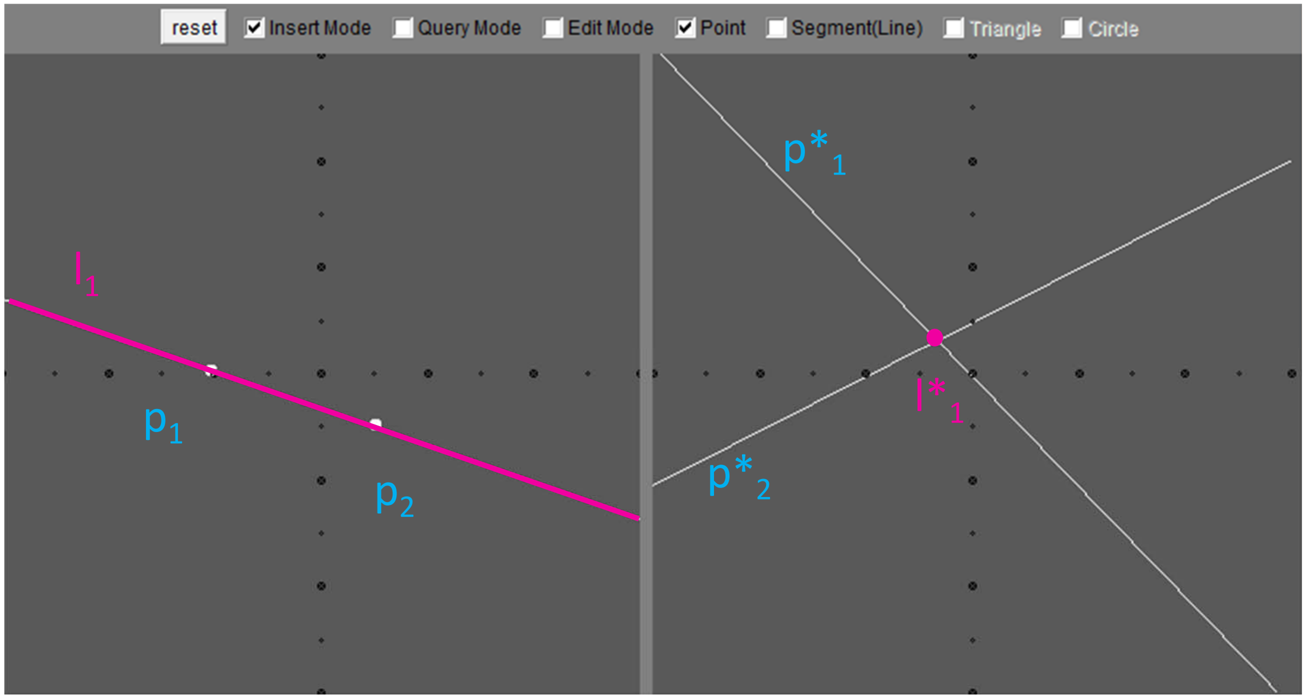
- $(p^*)^* = p$
- $p \in l \Leftrightarrow l^* \in p^*$ *incidence-preserving*
- p lies above $l \Leftrightarrow l^*$ lies above p^*

- $p_1 \in l_2 \Leftrightarrow l^*_2 \in p^*_1$
- p_3 is above $l_1 \Leftrightarrow l^*_1$ is above p^*_3

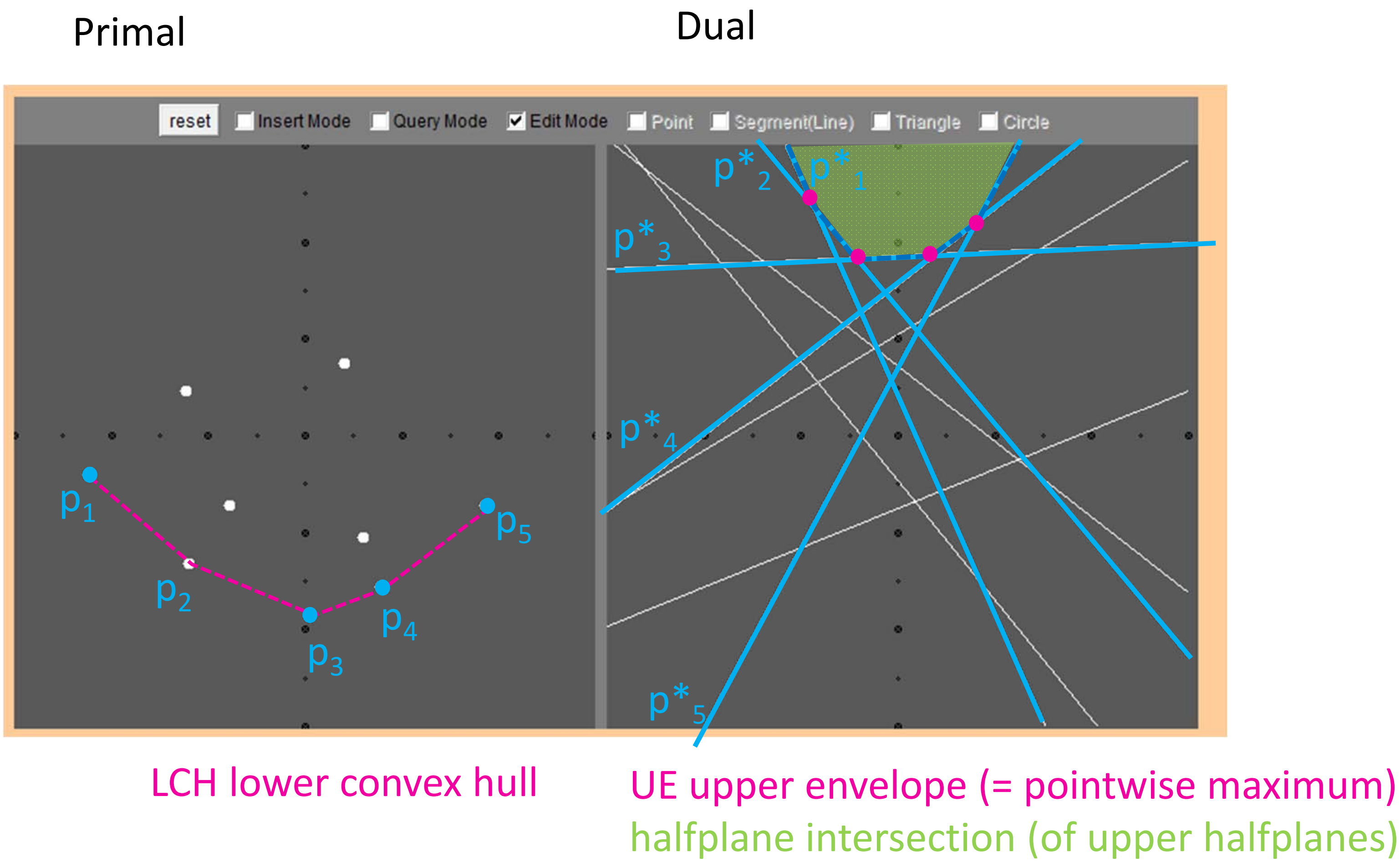
Properties:

Primal

Dual



LCH ≅ UE



- LCH = p_1, p_2, p_3, p_4, p_5
- UE = p^*_1, p^*_2, p^*_3, p^*_4, p^*_5