

# Point-Line Duality

Let  $P = \{p_1, \dots, p_n\} \subseteq \mathbb{R}^2$  be a set of  $n$  points. Now define a set  $P^* = \{p_1^*, \dots, p_n^*\}$  of  $n$  lines as follows:

## Primal plane

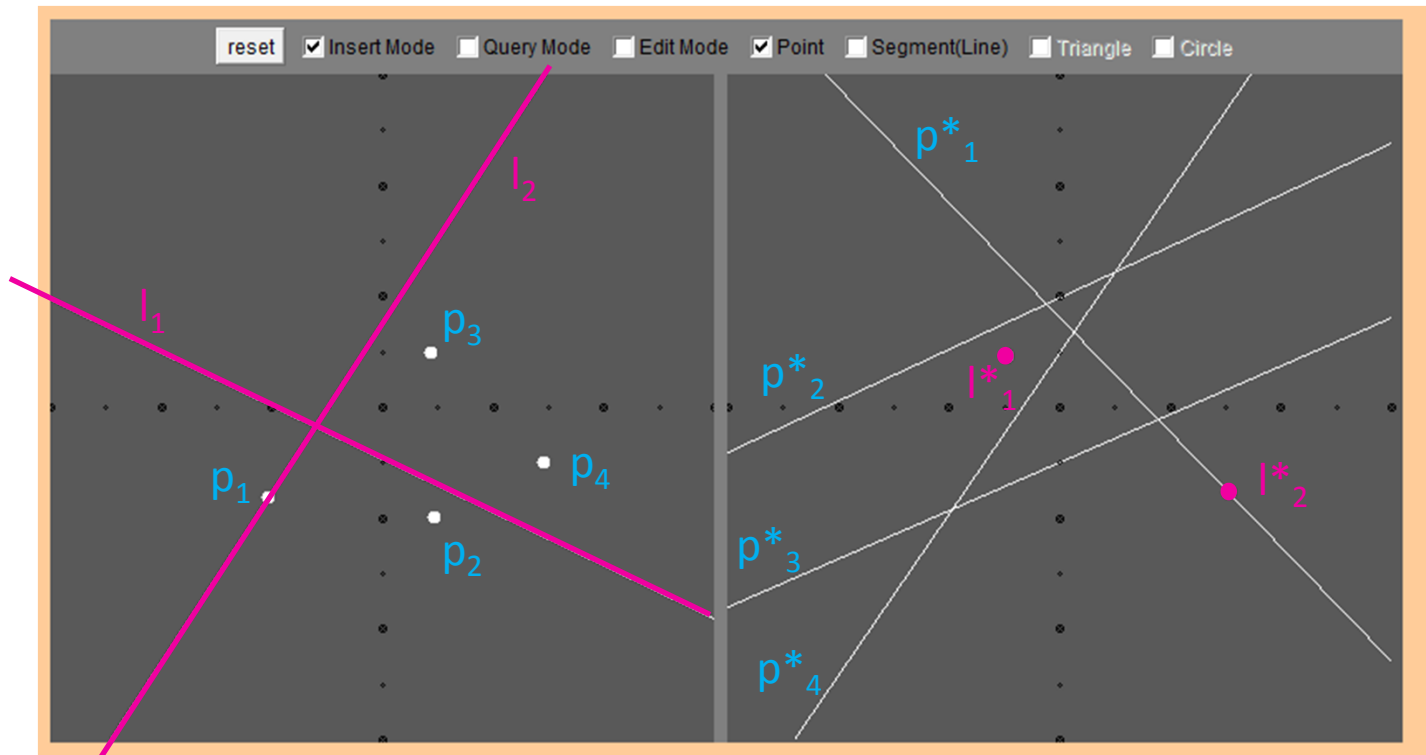
Point:  $p = (p_x, p_y)$

Line:  $l: y = mx + b$

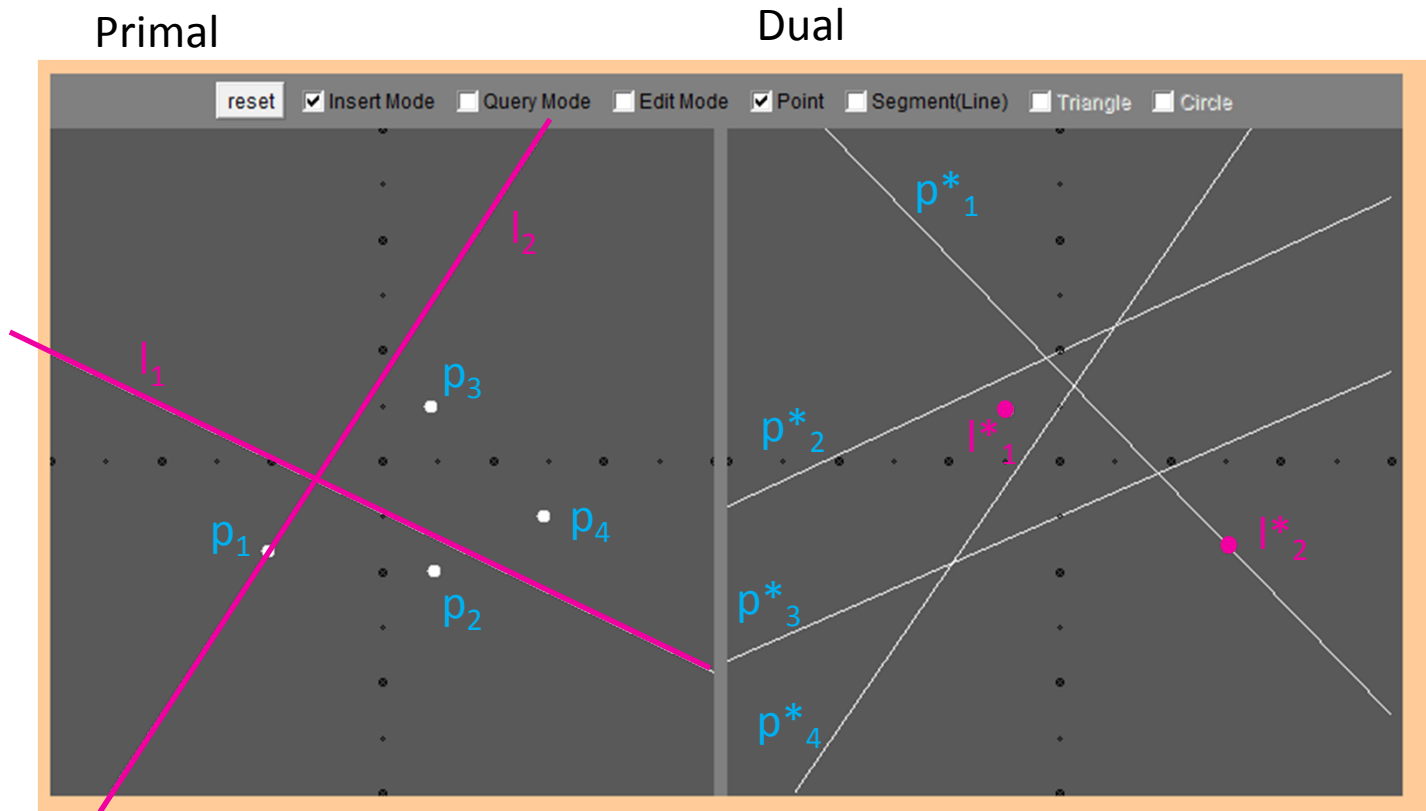
## Dual plane

Line:  $p^*: y = p_x x - p_y$

Point:  $l^* = (m, -b)$



# Properties:

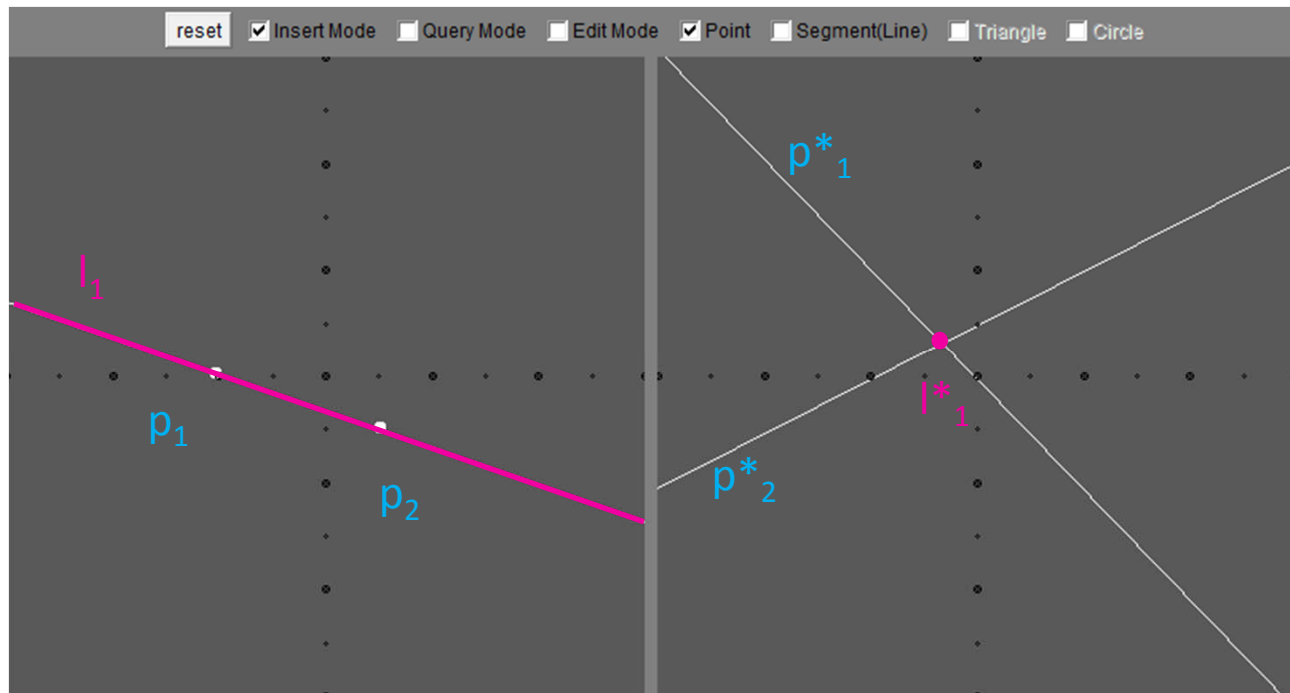


- $(p^*)^* = p$
- $p \in l \Leftrightarrow l^* \in p^*$  incidence-preserving
- $p$  lies above  $l \Leftrightarrow l^*$  lies above  $p^*$
- $p_1 \in l_2 \Leftrightarrow l_2^* \in p_1^*$
- $p_3$  is above  $l_1 \Leftrightarrow l_1^*$  is above  $p_3^*$

# Properties:

Primal

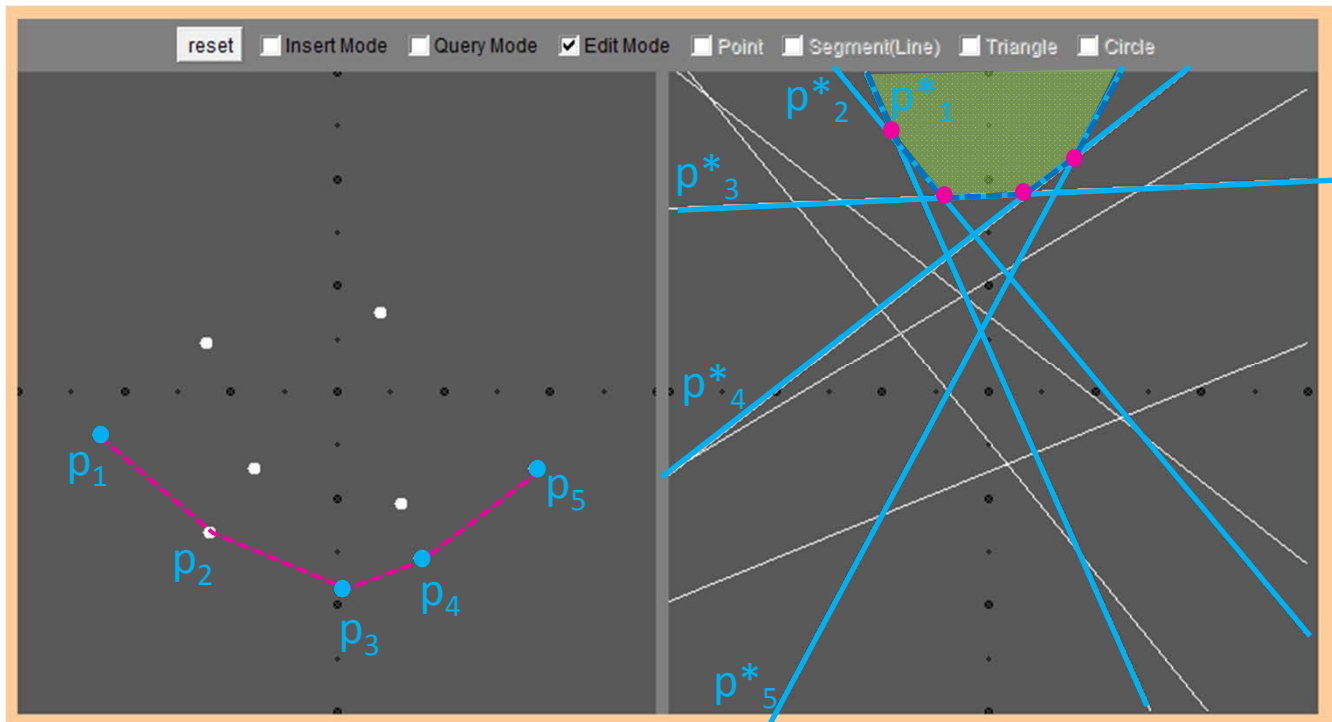
Dual



# LCH $\cong$ UE

Primal

Dual



LCH lower convex hull

UE upper envelope (= pointwise maximum)  
halfplane intersection (of upper halfplanes)

- LCH =  $p_1, p_2, p_3, p_4, p_5$
- UE =  $p^*_1, p^*_2, p^*_3, p^*_4, p^*_5$