

# Curvature Estimates of Point Clouds as a Tool in Quantitative Prostate Cancer Classification

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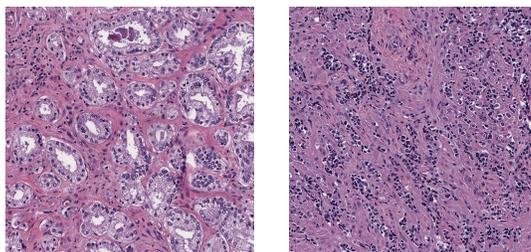
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## Abstract

Prostate cancer is one of the most commonly diagnosed cancers. Diagnosis involves several factors, including a pathologist grading biopsy and prostatectomy slides. The slides are 2D cross-sections of the biopsy or prostate. In such biopsies, glands appear as loops defined by the nuclei of cells defining the gland. As the cancer progresses, glands transform from circular to finger-like to unstructured. Currently, the severity of cancer (and by extension, treatment recommendations) is determined using the Gleason grading system, a visual analysis that compares biopsy features to a standard set of patterns in gland size, shape, and organization, and assigns biopsies scores on a scale of 1 through 5 [4]. In particular, pathologists analyze the appearance the tubelike glands of such cross sections. Typically, a less cancerous prostate has fairly circular tubelike glands, whereas a more cancerous prostate has less uniformly circular glands; see Figure 1 for examples.

Although the Gleason grading system is certainly helpful to patients and doctors, its qualitative nature has the potential to lead to inconsistencies in scores given to biopsies [2] [5]. Such inconsistencies motivate the goal of quantifying gland curvature to aid in developing a more consistent method of classifying prostate cancer. In this paper, we propose a method to describe the shape of glands using curvature.



**Figure 1** Example of stained cross sections of a needle core biopsy of prostate tissue. The dark purple dots correspond to nuclei and outline each tubular gland. The example on the left contains fairly uniformly curved tubular glands, and would not be classified as severely cancerous. The example on the right shows a more cancerous sample; in particular, we note that glands are beginning to lose their loop-like structure, and form sheets of cells.

**Methods** We focus on studying the curvature of glands. To use this in practice, we would need to take the following steps: (1) extract nuclei defining a single gland; (2) order the nuclei in a counter-clockwise loop around the gland; (3) define curvature for a discretized curve. We focus on step (3). We defer step (1) to the full paper and use the CRUST algorithm [1] for step (2).

The extrinsic curvature of an object embedded in space is defined by how much the object varies from a flat surface. For a smooth curve  $C$  embedded in  $\mathbb{R}^2$ , the curvature at a point  $x \in C$  is equal to  $1/r$ , where  $r$  is the radius of the circle that best approximates  $C$  at  $x$  [6]; the total



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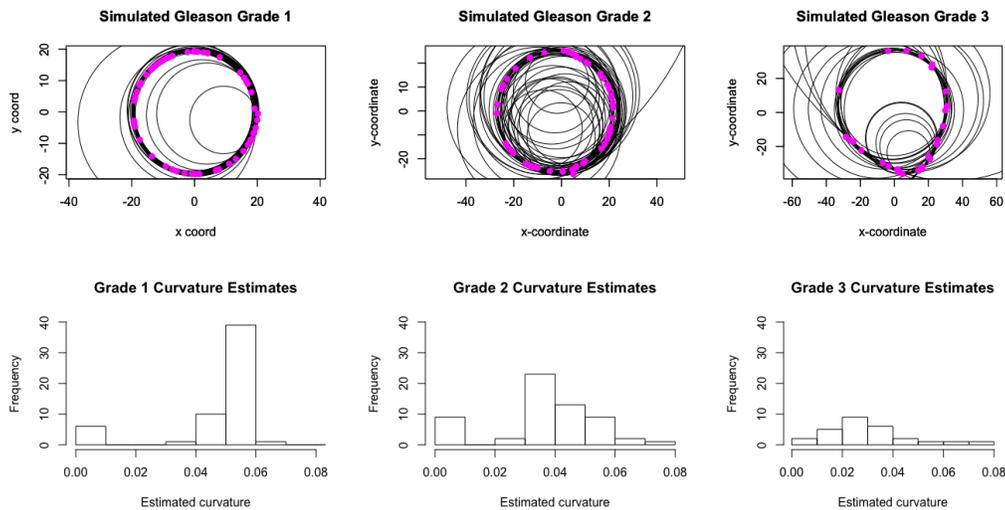
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curvature of a piecewise linear curve can be captured by turning angles [7]. We emulate the definition of curvature given above for smooth curves to estimate the curvature of prostate gland cross sections. For each nuclei  $n_i$ , we estimate the curvature by finding the best fitting circle containing  $n_i$  along with  $m$  neighbors on each side. Note that since a circle is defined by a minimum of three points, we require  $m \geq 1$ . Since we used the CRUST algorithm to find an ordering of our nuclei around a gland, these are the  $m$  nuclei before  $n_i$  and  $m$  nuclei after  $n_i$ . (If the number of nuclei is less than  $2m + 1$ , then we have duplicates in this set). Doing this for each nucleus, we obtain a distribution of local curvature estimates. This curvature distribution is our gland descriptor.

**Experimental Results** To test our method, we computed curvature distributions on simulated glands [3] for three different aggression levels; see Figure 2. As expected, preliminary results on simulated glands indicate that more cancerous glands tend to have higher variation in estimated local curvature than less cancerous glands.



■ **Figure 2** Curvature distributions for three simulated glands. The magenta dots in the top row are a simulation of the position of nuclei in glands. The best fitting circle for each nuclei using  $m$  neighbors on each side is shown (we used  $m = 2$ ). Curvature at a nuclei is then estimated as the reciprocal such a circle, the corresponding histogram of which is shown on the bottom row.

**Continued Research** We present the curvature distribution as a gland descriptor. The next step in this project is to study how the curvature distribution varies with Gleason grade and cancer severity using human biopsy data. Ultimately, we will use this descriptor in automated histology slide analysis.

**Keywords and phrases** curvature estimation, point clouds, prostate cancer

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