Homework 4 (30 Points)
Due 10/05/17 at the beginning of lab

1. (4 points) Determine whether each of these statements is true or false. Justify your answer shortly.
   a) $x \in \{x\}$
   b) $\{x\} \in \{x\}$
   c) $\{x\} \subseteq \{\{x\}\}$
   d) $\{x\} \subseteq \{x, \{x\}\}$

2. (4 points) Prove that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

3. (8 points) Let $A$ and $B$ be sets. Prove that
   (a) $A \cup (A \cap B) = A$
   (b) $A \cap (B - A) = \emptyset$

4. (6 points) For each of the following functions, determine whether it is injective and whether it is surjective, and if it is both injective and surjective, find the corresponding inverse function.
   (a) $f : \mathbb{R} \to \mathbb{R}, f(x) = \frac{1}{(x^2 + 1)}$
   (b) $f : \mathbb{R} \to \mathbb{R}, f(x) = -3x + 4$
   (c) $f : \mathbb{R}^+ \to \mathbb{R}^+, f(x) = \sqrt{x + 1}$

5. (4 points) Let $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2 + 3$ and $g : \mathbb{R}^+ \to \mathbb{R}^+$ with $g(x) = \sqrt{x}$ be two functions. Find $f \circ g$ and $g \circ f$.

6. (4 points) A function $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing iff $\forall x_1, x_2 \in \mathbb{R} : x_1 < x_2 \rightarrow f(x_1) < f(x_2)$. Prove that every strictly increasing function is one-to-one.