1. (4 points) Prove that $\sqrt{2}$ is irrational.

2. (4 points) Prove that there is no largest even positive integer.

3. (6 points) Show that these three statements are equivalent, where $a$ and $b$ are real numbers:
   (i) $a$ is less than $b$, (ii) the average of $a$ and $b$ is greater than $a$, and (iii) the average of $a$ and $b$ is less than $b$.

4. (6 points) Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$ for all $a, b, c \in \mathbb{R}$.

5. (4 points) Prove that there are no solutions in integers $x$ and $y$ to the equation $2x^2 + 5y^2 = 14$.

6. (6 points) Prove that for every rational number $a$ and every irrational number $b > a$, there is an irrational number $c \in (a, b)$. 