

Answer the questions in the space provided below. You may use the back of the page if you need more space. Time: 15 minutes.

Name and section: \_\_\_\_\_

1. Let  $S = \{f \mid f : \mathbb{R} \rightarrow \mathbb{R}\}$ . So,  $S$  is the set of all functions on the real numbers. We define a relation  $\sqsubseteq$  on  $S$  by:

$$f \sqsubseteq g \text{ iff } \forall x \in \mathbb{R} : f(x) \leq g(x)$$

- (a) Prove that the relation  $\sqsubseteq$  is reflexive. (20)
- (b) Prove that the relation  $\sqsubseteq$  is antisymmetric. (40)
- (c) Prove that the relation  $\sqsubseteq$  is transitive. (40)
- (d) **Bonus:** Is  $(S, \sqsubseteq)$  a totally ordered set? Justify your answer.