1. Consider the following recursive definition of a function $f : \mathbb{N} \rightarrow \mathbb{R}$

\[
\begin{align*}
&f(0) = \sqrt{2} \\
&f(1) = \sqrt{2} \\
&f(2) = \sqrt{2} \\
&f(n) = f(n-1) + f(n-2) - f(n-3), \text{ if } n \geq 3
\end{align*}
\]

Write down a closed-form definition for $f$ which is equivalent to the recursive one above (for example $g(x) = x + 3$). You do not have to justify your answer.

2. Recall that the factorial function is defined on positive integers by $n! = 1 \times 2 \times 3 \times \cdots \times n$. Prove by induction that for any $n \in \mathbb{N}$, such that $n \geq 1$, it is the case that $n! \leq n^n$. 