Answer the questions in the space provided below. You may use the back of the page if you need more space. Time: 15 minutes.

Name and section: _____

1. Consider the following recursive definition of a function $f : \mathbb{N} \to \mathbb{R}$ (50)

$$f(0) = \sqrt{2}$$
$$f(1) = \sqrt{2}$$
$$f(2) = \sqrt{2}$$
$$f(n) = f(n-1) + f(n-2) - f(n-3), \text{ if } n \geq 3$$

Write down a closed-form definition for f which is equivalent to the recursive one above (for example $g(x) = x + 3$). You do **not** have to justify your answer.

2. Recall that the factorial function is defined on positive integers by $n! = 1 \times 2 \times 3 \times \cdots \times n$. Prove by induction that for any $n \in \mathbb{N}$, such that $n \geq 1$, it is the case that $n! \leq n^n$. (50)