

Answer the questions in the space provided below. You may use the back of the page if you need more space. Time: 15 minutes.

Name and section: _____

1. For each of the functions below, determine whether it is an injection, surjection, bijection or neither of the three. You do **not** have to justify your answers.
 - (a) $f : \mathbb{N} \rightarrow \mathbb{Z}$, where $f(n) = n + 1$, for all $n \in \mathbb{N}$ (10)
 - (b) $g : \mathbb{R} \rightarrow \mathbb{R}$, where $g(x) = \frac{1}{x^2+1}$, for all $x \in \mathbb{R}$ (10)
 - (c) $h : \mathbb{R} \rightarrow \{0, 1\}$, where $h(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ 1 & \text{if } x < 0 \end{cases}$ (10)
 - (d) $k : \{1, 2, 3\} \rightarrow \{a, b, c\}$, where $k(1) = b$, $k(2) = a$, $k(3) = c$ (10)
 - (e) $s : A \rightarrow \bigcup_{a \in A} \{a\}$, where $s(a) = a$, for all $a \in A$ (10)

2. Recall that if A and B are both countable sets, then $A \cup B$ is also countable. Prove by induction that any finite union of countable sets is also countable. (50)