1. (Logic and Proof)
   (a) Prove that $\forall x \in \mathbb{R} \exists y \in \mathbb{R} \exists z \in \mathbb{R} : (x \neq 0) \rightarrow (y \times z > \frac{1}{x})$ (10)
   (b) Prove that $\exists x \in \mathbb{R} \forall y \in \mathbb{R} \forall z \in \mathbb{R} : (y - z \leq x) \lor (z - y \leq x)$ (10)

2. (Set Theory)
   Let $A$ be a finite set with $n \geq 1$ elements $A = \{A_1, A_2, \ldots, A_n\}$, where each $A_i$ is itself a set, such that for any $i, j$ we have $A_i \subseteq A_j$ or $A_j \subseteq A_i$.
   (a) Prove that $A$ contains an element $A_k$, such that $A_k$ is not a subset of any other element in $A$. (10)
   (b) Prove that $A_k$ is a superset of every element of $A$. (10)

3. (Induction and Recursion)
   Consider the function $f : \mathbb{N} \rightarrow \mathbb{N}$ recursively defined by:
   
   
   $f(0) = 0$
   $f(1) = 1$
   $f(2) = 2$
   $f(n) = (f(n - 3) + f(n - 2) + 3)f(n - 1)$

   Prove by induction that $f(n) \geq n$ for $n \in \mathbb{N}$.

4. (Relations)
   Consider the set $\mathbb{N}_\top = \mathbb{N} \cup \{\top\}$. We define a binary relation $\sqsubseteq$ on $\mathbb{N}_\top$ by:

   
   $n \sqsubseteq m$ iff $m = \top$ or $n \leq m$

   where $n \leq m$ is the standard order on $\mathbb{N}$.
   (a) Prove that $(\mathbb{N}_\top, \sqsubseteq)$ is a poset. (10)
   (b) Prove that any non-empty subset of $\mathbb{N}_\top$ has a least upper bound in $\mathbb{N}_\top$. (10)

5. (Graphs)
   Let $G$ be a graph which contains a simple circuit of odd length. Prove that $\chi(G) \geq 3$. (20)