1. **(Logic and Proof)**
   (a) Prove that $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ \exists z \in \mathbb{R} : (x \neq 0) \rightarrow (y \times z > \frac{1}{x})$ (10)

   (b) Prove that $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ \forall z \in \mathbb{R} : (y - z \leq x) \vee (z - y \leq x)$ (10)

2. **(Set Theory)**
   Let A be a finite set with $n \geq 1$ elements $A = \{A_1, A_2, \ldots, A_n\}$, where each A_i is itself a set, such that for any i, j we have $A_i \subseteq A_j$ or $A_j \subseteq A_i$.

   (a) Prove that A contains an element A_k, such that A_k is not a subset of any other element in A. (10)

   (b) Prove that A_k is a superset of every element of A. (10)

3. **(Induction and Recursion)** (20)
   Consider the function $f : \mathbb{N} \rightarrow \mathbb{N}$ recursively defined by:

$$
\begin{aligned}
f(0) &= 0 \\
f(1) &= 1 \\
f(2) &= 2 \\
f(n) &= (f(n-3) + f(n-2) + 3)f(n-1)
\end{aligned}
$$

   Prove by induction that $f(n) \geq n$ for $n \in \mathbb{N}$.

4. **(Relations)**
   Consider the set $\mathbb{N}_\top = \mathbb{N} \cup \{\top\}$. We define a binary relation $\sqsubseteq$ on $\mathbb{N}_\top$ by:

$$n \sqsubseteq m \text{ iff } m = \top \text{ or } n \leq m$$

   where $n \leq m$ is the standard order on $\mathbb{N}$.

   (a) Prove that $(\mathbb{N}_\top, \sqsubseteq)$ is a poset. (10)

   (b) Prove that any non-empty subset of $\mathbb{N}_\top$ has a least upper bound in $\mathbb{N}_\top$. (10)

5. **(Graphs)** (20)
   Let G be a graph which contains a simple circuit of odd length. Prove that $\chi(G) \geq 3$.