1. **(Logic and Proof)**
   (a) Show that the following propositional sentence: (5)

$$((a \vee \neg a) \to b) \wedge (b \wedge c \to d) \wedge (e \vee \neg f)$$

   is satisfiable.
   (b) Prove that $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ \exists z \in \mathbb{R} : (x > 0) \to (x = y^2 \wedge x = z^2 \wedge y \neq z)$ (10)
   (c) Prove that $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ \forall z \in \mathbb{R} : (y^2 + z^2 = x^2) \to (y = 0 \wedge z = 0)$ (10)

2. **(Set Theory)**
   Let A be an arbitrary set and let $\mathcal{P}(A)$ be its powerset. Consider the set $S = \bigcup_{X \in \mathcal{P}(A)} X$, which is the union of all subsets of A.
   (a) Prove that $S \subseteq A$. (10)
   (b) Prove that $A \subseteq S$. (10)
   (c) Show that $A = S$. (5)

3. **(Induction and Recursion)**
   Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by:

$$f(n) = n^{(n-1)^{(n-2)^{(n-3)^{\cdot^{\cdot^{\cdot^{2^{1^{0}}}}}}}}}$$

   which computes a tower of decreasing powers of n in a right-associative way. That is, the powers are computed in a right-to-left or top-to-bottom way. So, $f(4) = 4^{3^{2^{1^{0}}}} = 4^{3^{2^{1}}} = 4^{3^{2}} = 4^{9}$.
   (a) Provide a recursive definition for the function f. You do **not** have to prove that your definition is correct. (10)
   (b) Prove by induction that $f(n) \geq n$ for each $n \in \mathbb{N}$. (15)

4. **(Combinatorics)**
   Answer each of the following questions. You do **not** have to provide any proofs.
   (a) Let A be a set with 13 elements. How many subsets of A are there with at least 3 elements and at most 7 elements? (5)
   (b) Let $B = \{a, b, c, d, e, f\}$. How many words can we form with the letters of B, where each letter is used exactly once and one of the letters in $\{a, b, c\}$ is next to the other two? (10)
   (c) Let C be a set with three elements given by $C = \{a, b, c\}$. How many functions $f : C \to \mathbb{N}$ are there, such that $f(a) + f(b) + f(c) = 7$? (10)