1. (Logic and Proof)
   (a) Show that the following propositional sentence:
   
   \[ ((a \lor \neg a) \rightarrow b) \land (b \land c \rightarrow d) \land (c \lor \neg f) \]

   is satisfiable.
   (b) Prove that \( \forall x \in \mathbb{R} \exists y \in \mathbb{R} \exists z \in \mathbb{R} : (x > 0) \rightarrow (x = y^2 \land x = z^2 \land y \neq z) \)
   (c) Prove that \( \exists x \in \mathbb{R} \forall y \in \mathbb{R} \forall z \in \mathbb{R} : (y^2 + z^2 = x^2) \rightarrow (y = 0 \land z = 0) \)

2. (Set Theory)
   Let \( A \) be an arbitrary set and let \( \mathcal{P}(A) \) be its powerset. Consider the set \( S = \bigcup_{X \in \mathcal{P}(A)} X \), which is the union of all subsets of \( A \).
   (a) Prove that \( S \subseteq A \).
   (b) Prove that \( A \subseteq S \).
   (c) Show that \( A = S \).

3. (Induction and Recursion)
   Consider the function \( f : \mathbb{N} \rightarrow \mathbb{N} \) defined by:
   
   \[ f(n) = n^{(n-1)(n-2)(n-3)} \cdot 2^{10} \]

   which computes a tower of decreasing powers of \( n \) in a right-associative way. That is, the powers are computed in a right-to-left or top-to-bottom way. So, \( f(4) = 4^{3^2} = 4^9 = 4^8 = 4^9 \).
   (a) Provide a recursive definition for the function \( f \). You do not have to prove that your definition is correct.
   (b) Prove by induction that \( f(n) \geq n \) for each \( n \in \mathbb{N} \).

4. (Combinatorics)
   Answer each of the following questions. You do not have to provide any proofs.
   (a) Let \( A \) be a set with 13 elements. How many subsets of \( A \) are there with at least 3 elements and at most 7 elements?
   (b) Let \( B = \{a, b, c, d, e, f\} \). How many words can we form with the letters of \( B \), where each letter is used exactly once and one of the letters in \( \{a, b, c\} \) is next to the other two?
   (c) Let \( C \) be a set with three elements given by \( C = \{a, b, c\} \). How many functions \( f : C \rightarrow \mathbb{N} \) are there, such that \( f(a) + f(b) + f(c) = 7 \)?