CMPS 3130/6130 Computational Geometry – Spring 20

Final Exam Review

Relevant Material

- All material from 1/13/20 until 4/16/20 (inclusive)
- This includes homeworks 1–9
- All slides, board pictures, and Miro boards
- The covered chapters in the book are listed below under "Ch.", and the covered chapters in Mount's lecture notes are listed below under "L."
- Need to know basic ideas and complexities (runtimes, space) of algorithms and data structures. (Distinguish between deterministic and expected runtimes.)
- **Convex Hulls (Ch. 1, L. 1,3)**
  - Practice problems from the book: Ch. 1, page 15; 1.1–1.3, 1.5, 1.6a, 1.7, 1.8
  - Definition of convex set, and of convex hull of a set of points
  - Orientation test (halfplane test)
  - Algorithms: Jarvis' March, Incremental Insertion, Divide and Conquer, Graham's Scan
  - $\Omega(n \log n)$ lower bound by reduction from sorting
  - Graduate: Chan's algorithm
- **Sweepline Algorithms (Ch. 2, 5.1; L. 5**
  - Practice problems from the book: Ch. 2, page 41; 2.1, 2.2, 2.5–2.10
  - Structure of a sweepline algorithm: Cleanliness property, sweep line status, event queue, event handling
  - Closest pair
  - Line segment intersection (output-sensitive algorithm)
- **Triangulation and Guarding (Ch. 3; L. 6, 22**
  - Practice problems from the book: Ch. 3, page 60; 3.1–3.7, 3.11–3.14
  - Triangulation of simple polygons:
    - #triangles, dual graph, 3-coloring lemma, art-gallery theorem
    - $O(n^2)$ algorithm (D&C based on ear cutting / proof of theorem 1)
    - $O(n)$ algorithm for monotone polygons (definition of a monotone polygon)
    - $O(n \log n)$ sweep-line algorithm for non-monotone polygons: Split into monotone pieces, then triangulate each monotone piece. (It is not necessary to know all the cases of this algorithm by heart.)

- **Point Location (Ch. 6.1, 6.2; L. 25, 9, 10**
  - Practice problems from the book: Ch. 6, page 144; 6.1, 6.4–6.8, 6.13
  - Planar subdivision (definition, complexity), Euler's formula, doubly-connected edge list
  - Slab method for point location
  - Kirkpatrick's algorithm: Build hierarchy by incrementally removing independent set of points and retriangulating
  - Trapezoidal map; randomized incremental construction (need to know results, not details of analysis); backwards analysis, DAG for point location
- **Voronoi Diagrams (Ch. 7.1, 7.2; L. 11)**
  - Practice problems from the book: Ch. 7, page 170; 7.1–7.3, 7.5–7.7, 7.12
  - Definition, bisectors, complexity; applications
  - Fortune's sweep
    - Beach line consisting of sequence of parabolas stored implicitly in binary search tree
    - Site events, circle events
- **Delaunay Triangulation (Ch. 9.1, 9.2; L. 12, 13**
  - Practice problems from the book: Ch. 9, page 215: 9.2-9.3, 9.8, 9.9, 9.12–9.16
  - Triangulations of point sets
  - DT dual graph of VD
  - Different characterizations of DT (empty circumcircle, legal edges, angle-optimality)
  - Empty circle property
  - Edge flips, and edge-flip based algorithm to compute DT
  - Randomized incremental construction
  - Connection between DT and 3D CH
- **Linear Programming (Ch. 4.2-4.4; L. 8)**
  - Practice problems from the book: Ch. 4, page 91; 4.9-4.11, 4.14-4.16
  - LP definition
  - Halfplane (or halfspace) intersection; relationship to LP
  - Randomized incremental construction (RIC), uses backwards analysis
- **Point-Line Duality and Arrangements (Ch. 8.2, 8.3; L. 14, 15, 28)**
  - Practice problems from the book: Ch. 8, page 188; 8.1–8.11
  - Point-line duality definition; dual point, lines, line segments
  - Arrangements of lines
  - Zone theorem

- Ham sandwich theorem
- Interconnectedness between CH-DT-VD-UE

- **Orthogonal range searching (Ch. 5.1–5.4; L. 31–34)**
  - Practice problems from the book: Ch. 5, page 117; 5.1–5.7, 5.10, 5.11
  - Range trees, nested range trees
  - Grad: Fractional cascading
  - kd-trees
  - Windowing: interval trees, segment trees, windowing (range-segment trees)

Not on the Test

- Details on linear algebra
- Detailed analyses of randomized algorithms
- Topics of paper presentations
- Robot motion planning
- Grad: Dobkin-Kirkpatrick hierarchy
- Grad: Weighted Voronoi diagrams