CMPS 3130 Computational Geometry – Spring 20

3/26/20

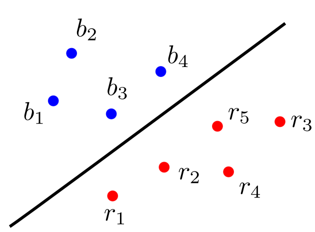
8. Homework (undergrad)

Due **4/2/20** before class

Please justify all your answers. Often it helps to draw pictures.

1. **Dual Line Segment (9 points)**
   Consider two points $a = (0, -2)$ and $b = (3, 1)$. Let l be the line through a and b.
   (a) (2 points) Draw the primal plane with a, b, l and draw the dual plane with a^*, b^*, l^*. Specify the equations for l, a^*, b^* and the coordinates for l^*.
   (b) (2 point) Consider the points $c = (1, -1)$ and $d = (2, 0)$. Draw c^* and d^* in the dual plane.
   (c) (3 points) Now consider the line segment s from a to b. (Note that c and d lie on s.) What is the dual of s? You can describe it in words.
   (d) (2 points) If a line l_2 in the primal plane intersects s, where must its dual point l_2^* lie?

2. **Linear Separator (8 points)**
   Let $R = \{r_1, \ldots, r_m\}$ be set of m red points and let $B = \{b_1, \ldots, b_n\}$ be a set of n blue points in the plane. A line l is called a **linear separator** if all points of R lie on one side of l and all points of B lie on the other side. (You may assume appropriate general position, and may disregard points that lie exactly on the line.)



   Use point-line duality to develop an algorithm for this problem which runs in expected linear time. *(Hint: Linear Programming.)*

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3. **Sign Vectors (8 points)**

Consider an arrangement $\mathcal{A}$ of six lines $l_1, l_2, l_3, l_4, l_5, l_6$, and let f be an arbitrary vertex, edge, or face of $\mathcal{A}$. Then f has an associated *sign vector* $(s_1, s_2, s_3, s_4, s_5, s_6)$, where for each $1 \leq i \leq 6$:

$$s_i = \begin{cases} +1, & \text{if } f \text{ lies above } l_i \\ 0, & \text{if } f \text{ lies on } l_i \\ -1, & \text{if } f \text{ lies below } l_i \end{cases}$$

(a) For each of the sign vectors below, give an arrangement of six lines that has a vertex, edge, or face with this sign vector. Label the lines and mark the vertex, edge, or face. Make the arrangement simple, if possible, or argue why the arrangement cannot be simple.

i. $(+1, +1, +1, +1, +1, +1)$ ii. $(+1, 0, 0, -1, -1, -1)$ iii. $(-1, 0, 0, -1, +1, -1)$

(b) Can one find a single arrangement of lines that contains a vertex, edge, or face for each of the three sign vectors in (a)? Argue why or why not.