CMPS 3130 Computational Geometry – Spring 20

3/12/20

7. Homework (undergrad)

Due **3/26/20** before class

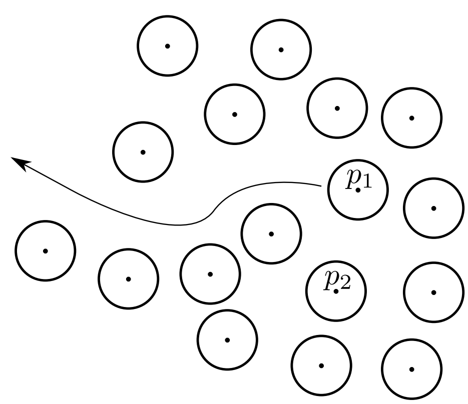
Please justify all your answers. Often it helps to draw pictures.

1. **Maintenance of Delaunay Triangulations (10 points)**
   Let P be a set of n points in the plane, for which a Delaunay Triangulation has already been constructed using randomized incremental construction. You are given all data structures that were used to compute the Delaunay Triangulation.

   (a) (5 points) Given a point $q \in P$, give an algorithm that *deletes* q from the Delaunay Triangulation, i.e., it computes the $DT(P \setminus \{q\})$. Analyze the runtime. The runtime should depend on $k = \deg(q, DT(P))$.

   (b) (5 points) Given another point $q \notin P$, give an algorithm that *adds* q to the Delaunay Triangulation, i.e., it computes $DT(P \cup \{q\})$. What is the runtime, expressed in n and k, where $k = \deg(q, DT(P \cup \{q\}))$?

2. **Disk Escape (10 points)**
   Assume you are given a set of n non-intersecting unit disks in the plane with center points $p_1, \ldots, p_n$. Determine whether it is possible for the disk with center point p_1 to *escape* from the others. The disk can escape iff it can continuously move arbitrary far away from the other disks without shifting or intersecting the other disks. In the example below, the disk with center point p_1 can escape, but the disk with center point p_2 cannot.



   Give an $O(n \log n)$-time algorithm to solve this problem. Your algorithm should use a Voronoi diagram. Either indicate that the disk cannot escape, or output a path along which to move the center point of the disk.

3. **Railway Tracks (10 points)**
   On n parallel railway tracks n trains are going with constant speeds $v_1, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, \ldots, k_n$.

   Give an $O(n \log n)$ time algorithm, based on halfplane intersection, that detects all trains that at some moment in time are leading. **Analyze the runtime.**