CMPS 3130 Computational Geometry – Spring 20

3/5/20

# 6. Homework (undergrad)

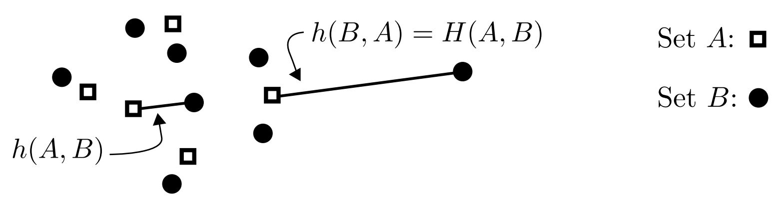
Due **3/12/20** before class

**Please justify all your answers. Often it helps to draw pictures.**

1. **Delaunay (5 points)**
   Sketch a deterministic (i.e., non-randomized) algorithm to compute the Delaunay Triangulation of a point set $P$ of $n$ points in $O(n \log n)$ time. Analyze its runtime.

2. **Parabolic Arc (5 points)**
   Give an example where the parabola defined by some site $p_i$ contributes $O(n)$ arcs to the beach line, where $n$ is the number of input points.

3. **Hausdorff Distance (8 points)**
   Let $A$ and $B$ be two point sets in the plane with $m = |A|$ and $n = |B|$. The *directed Hausdorff distance* $h(A, B)$ is defined as $h(A, B) = \max_{a \in A} \min_{b \in B} d(a, b)$, where $d(.,.)$ is the Euclidean distance. The *(undirected) Hausdorff distance* $H(A, B)$ is defined as $H(A, B) = \max\{h(A, B), h(B, A)\}$.

   Use the Voronoi diagram and point location structures to show that the undirected Hausdorff distance can be computed in $O((n + m) \log(n + m))$ time.

4. **Sum of Edge Lengths (5 points)**
   It appears that illegal edges are often long edges, so it is a natural question to ask whether the Delaunay triangulation might minimize edge lengths. Give an example which shows that the Delaunay triangulation of a point set is not always the triangulation with the minimum sum of edge lengths.