1/23/20

2. Homework (undergrad)

Due **1/30/20** before class

1. **Line segment intersection (10 points)**
   Given two line segments $\overline{ab}$ and $\overline{cd}$ in the plane, where $a, b, c, d \in \mathbb{R}^2$. The goal is to test them for intersection.

   (a) (3 points) Let $a = \binom{6}{5}$, $b = \binom{14}{9}$, $c = \binom{7}{2}$, and $d = \binom{9}{10}$. Express each line segment as a convex combination, and use this representation to determine if $\overline{ab}$ and $\overline{cd}$ intersect, and if so, compute their intersection point.

   (b) (2 points) Do $\overline{eb}$ and $\overline{cd}$ intersect, where $e = \binom{10}{7}$? What is different compared to part (a)?

   (c) (5 points) Explain how you can use one or more orientation tests to test if two line segments intersect. *(Hint: Case analysis. Draw pictures of examples, and determine important configurations of a, b, c, d.)*

2. **Lower bounds (3 points)**
   Prove a lower bound of $\Omega(n \log n)$ for SORTING, by reducing from ELEMENT UNIQUENESS (i.e., by using the knowledge that ELEMENT UNIQUENESS has a lower bound of $\Omega(n \log n)$).

3. **Visible Segments Sweep (10 points)**
   Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We say that the point p *sees* a line segment s if there is a point $q \in s$ such that the segment pq does not intersect any other line segment of S. We wish to determine all line segments of S that p can see.
   Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p to sweep the plane. You do not have to give pseudocode but you should explain all the necessary components of the sweep.