CMPS 6130 Computational Geometry – Spring 20

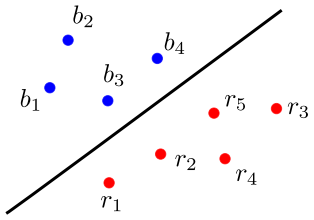
3/26/20

8. Homework (grad)

Due **4/2/20** before class

Please justify all your answers. Often it helps to draw pictures.

1. **Linear Separator (8 points)**
   Let $R = \{r_1, \ldots, r_m\}$ be set of m red points and let $B = \{b_1, \ldots, b_n\}$ be a set of n blue points in the plane. A line l is called a **linear separator** if all points of R lie on one side of l and all points of B lie on the other side. (You may assume appropriate general position, and may disregard points that lie exactly on the line.)



   Use point-line duality to develop an algorithm for this problem which runs in expected linear time. *(Hint: Linear Programming.)*

2. **Dual Line Segment and Triangle (9 points)**
   (a) (3 points) What is the dual of a line segment? You can describe it in words.
   (b) (3 points) Given a line segment s and a line l. If l intersects in the primal plane, where must its dual point l^* lie?
   (c) (3 points) Consider a (solid) triangle Δpqr with corner points p, q, r. Describe its dual.

3. **Convex Hull of Intersections (8 points)**
   Let $\mathcal{L}$ be a set of n lines in the plane, no two of which are parallel. Let S be the set of all $O(n^2)$ intersection points between any two lines in $\mathcal{L}$. Give an $O(n \log n)$ time algorithm to compute an axis-parallel rectangle that contains S.

   (Hint: Your algorithm cannot compute all points in S explicitly. Sort all lines by slope, and prove that it is enough to consider only a certain subset of intersection points.)