CMPS 6130 Computational Geometry – Spring 20

3/12/20

7. Homework (grad)

Due **3/26/20** before class

Please justify all your answers. Often it helps to draw pictures.

1. **Star-Shaped Polygon (10 points)**
   A simple polygon P is called *star-shaped*, if it contains a point c such that for any point $p \in P$ the line segment $\overline{cp}$ is contained in P.

   Use an LP-based approach to develop an algorithm to decide whether a simple polygon is star-shaped. Make your algorithm **as efficient as possible.** (Can you make it run in expected linear time?)

2. **Railway Tracks (10 points)**
   On n parallel railway tracks n trains are going with constant speeds $v_1, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, \ldots, k_n$.

   Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading. To this end, use the algorithm for computing the intersection of half-planes.

3. **Weighted Voronoi Diagrams (10 points)**
   Let $P = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}^2$, and let $w_i > 0$ be the *weight* of point site p_i, for each $i = 1, \ldots, n$. In the *additively weighted* Voronoi diagram the Voronoi cell for p_i is defined as

   $$V_{add}(p_i) = \{q \in \mathbb{R}^2 \mid w_i + ||p_i - q|| < w_j + ||p_j - q|| \text{ for all } p_j \in P \setminus \{p_i\}\}$$

   In the *multiplicatively weighted* Voronoi diagram the Voronoi cell for p_i is defined as

   $$V_{mult}(p_i) = \{q \in \mathbb{R}^2 \mid w_i * ||p_i - q|| < w_j * ||p_j - q|| \text{ for all } p_j \in P \setminus \{p_i\}\}$$

   Show how the bisectors look like for both kinds of weighted Voronoi diagrams, and give some examples of Voronoi diagrams for each case. You are welcome to research on the web as long as you give references.