3/5/20

6. Homework (grad)

Due **3/12/20** before class

Please justify all your answers. Often it helps to draw pictures.

1. **Gabriel Graph (9 points)**
   Let P be a set of n points in the plane. The *Gabriel graph* $GG(P)$ is defined as follows: Two points $p, q \in P$ are connected by an edge in $GG(P)$ iff the circle with diameter pq does not contain any other point of P in its interior.

   (a) Prove that $DT(P)$ contains $GG(P)$. I.e., every edge in $GG(P)$ is also a Delaunay edge.

   (b) Prove that p and q are adjacent in $GG(P)$ if and only if the Delaunay edge between p and q intersects its dual Voronoi edge.

2. **Worst-Case DT Runtime (7 points)**
   The randomized incremental construction of the Delaunay triangulation of a set of n points in the plane takes $\Omega(n^2)$ time in the worst-case. That means that, for each n, there is a set of n points together with a particular input order such that the algorithm executes $\Omega(n^2)$ edge flips.

   What properties are required to cause that many flips? Please sketch the construction of such a bad input (it should work for general n). *(Hint: It might help to play the Voronoi/Delaunay demo linked to from the resources website, or with Voroglide (on Canvas).)*

3. **Reverse Voronoi (10 points)**
   Suppose we are given a subdivision of the plane into n convex regions. We suspect that this subdivision is a Voronoi diagram, but we do not know the sites. Develop an algorithm that finds a set of n point sites whose Voronoi diagram is exactly the given subdivision, if such a set exists.
   Hint: First consider a Voronoi diagram for three vertices. For the general case, if you are given the position of one point, how can you compute the position of the other points?