2/6/20

4. Homework (grad)

Due **2/13/20** before class

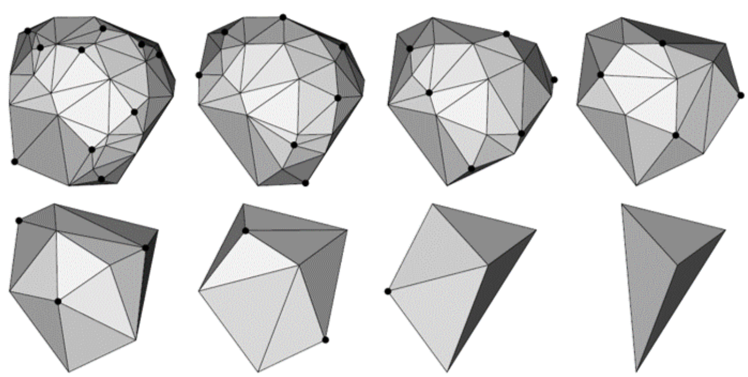
Please justify all your answers. Often it helps to draw pictures.

1. **DCEL (8 points)**
   (a) Which of the following equalities are always true? Justify your answers.
      i. $Twin(Twin(\vec{e})) = \vec{e}$
      ii. $Next(Prev(\vec{e})) = \vec{e}$
      iii. $Twin(Prev(Twin(\vec{e}))) = Next(\vec{e})$
   (b) You are given a planar subdivision in a doubly-connected edge list, where $Twin(\vec{e}) = Next(\vec{e})$ for every half-edge $\vec{e}$. How many faces can the subdivision have?

2. **Adjacent Vertices (10 points)**
   You are given a planar subdivision in a doubly-connected edge list, and a vertex v in this DCEL. Give pseudocode to output all vertices adjacent to v in *clockwise* order. Your algorithm should run in $O(deg(v))$ time, where $deg(v)$ is the degree of v. *(Hint: Draw an example picture and run your algorithm on this example to make sure it works.)*

3. **Dobkin-Kirkpatrick (8 points)**
   Let P be a convex polytope with n vertices in $\mathbb{R}^3$. Such a convex polytope is defined as the convex hull of these vertices, and its boundary is a connected planar embedded graph.

   The Kirkpatrick hierarchy can also also be used to create a hierarchy of polytopes: The Dobkin-Kirkpatrick hierarchy. The preprocessing time, space complexity, and query time are the same as for Kirkpatrick's hierarchy. See below for an example sequence of polytopes from such a hierarchy (the corresponding DAG is not shown):



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Assume this hierarchy has been computed for P. Describe and analyze an algorithm that for a given query point $q \in \mathbb{R}^3$ computes the distance from q to P, i.e., the smallest distance from Q to any point in P, in $O(\log n)$ time.

(You can assume the existence of basic geometric primitives, such as distance computation from a point to a plane or to a triangle. Note that P is solid, so if q lies inside P then the distance is zero.)