CMPS 6130 Computational Geometry – Spring 20

1/23/20

2. Homework (grad)

Due **1/30/20** before class

1. **Line segment intersection (5 points)**
Explain how you can use one or more orientation tests to test if two line segments $\overline{ab}$ and $\overline{cd}$ intersect, where $a, b, c, d \in \mathbb{R}^2$. *(Hint: Case analysis. Draw pictures of examples, and determine important configurations of a, b, c, d.)*

2. **Lower bounds (9 points)**
Consider the following problems:

   Sorting: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, output the same numbers in non-decreasing order.

   Element Uniqueness: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, are there i, j, with $i \neq j$, such that $x_i = x_j$?

   Closest Pair: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$, output the closest pair of points in P.

   All Nearest Neighbors: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$. Compute for each point in P its *nearest neighbor* in P (i.e., point at minimum distance).

   (a) Prove a lower bound of $\Omega(n \log n)$ for Sorting, by reducing from Element Uniqueness (i.e., by using the knowledge that element Uniqueness has a lower bound of $\Omega(n \log n)$).
   (b) Prove a lower bound of $\Omega(n \log n)$ for Closest Pair by reducing from an appropriate problem.
   (c) Prove a lower bound of $\Omega(n \log n)$ for All Nearest Neighbors by reducing from an appropriate problem.

3. **Visible Segments Sweep (10 points)**
Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We say that the point p *sees* a line segment s if there is a point $q \in s$ such that the segment pq does not intersect any other line segment of S. We wish to determine all line segments of S that p can see.
Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p to sweep the plane. You do not have to give pseudocode but you should explain all the necessary components of the sweep.