CMPS 6130 Computational Geometry – Spring 20

1/16/20

1. Homework (grad)

Due **1/23/20** before class

1. **Binary search (10 points)**
   Assume you have an orientation test available which can determine in constant time whether three points make a left turn (i.e., the third point lies on the left of the oriented line described by the first two points) or a right turn. Now, let a point q and a convex polygon $P = \{p_1, \ldots, p_n\}$ in the plane be given, where the points of P are stored in an array in counter-clockwise order around P. Give pseudo-code to determine an upper tangent from q to P in $O(\log n)$ time, and analyze its runtime. *(Hint: It helps to annotate your code with pictures.)*

2. **Convex hull of line segments (10 points)**
   Let S be a set of n line segments in the plane. Let P be the set of $2n$ endpoints of the segments in S. Prove that the convex hull of S is exactly the same as the convex hull of P. *(Hint: It might help to break the proof into two parts, $CH(S) \subseteq CH(P)$ and $CH(P) \subseteq CH(S)$.)*

3. **Reading+: Chan's convex hull (10 points)**
   Please read Chan's algorithm for computing the convex hull in the plane in $O(n \log h)$ time, as described in lecture 19 in Mount's notes (page 19-23 top). Note that the LiveCG jar file (see link on Resources page) contains a demo of Chan's algorithm. Please feel free to post questions on Piazza.

   Consider the following formulation of Chan's main algorithm:

   (1) $h^* = 2$; $L =$ fail
   (2) while ($L \neq$ fail)
      (a) $h^* = \min(2^{2^i}, n)$
      (b) $L =$RestrictedHull(P, h^*)
      (c) i++
   (3) return L

   Let h be the number of vertices on the convex hull of P. If $h \leq h^*$ then RestrictedHull(P, h^*) returns the convex hull of P, otherwise it returns "fail".

   For each of the two cases below, determine the big-Oh runtime of Chan's algorithm when replacing line (2)(a) with the shown expression. Justify your answers.

   (a) $h^* = \min(i^2, n)$
   (b) $h^* = \min(2^{2^{2^i}}, n)$