CMPS 2200 Introduction to Algorithms – Fall 17

11/30/17

Lab Worksheet

1. **Decision tree**
   Draw the decision tree for Mergesort for an array $A[0..2]$ of $n = 3$ elements. Note that the first split is $A[0] : A[1..2]$. Annotate the decision tree with comments indicating the part of the algorithm that a comparison belongs to.

2. **Random permutation**

   (a) Give pseudocode for an efficient algorithm that computes a random permutation of an array of n distinct numbers. What is the runtime of your algorithm? (Can you make the algorithm be in-place, i.e., using only additional constant space?)

   (b) Assume you are given an array $A[0..n-1]$ of n distinct numbers, and you compute a random permutation of it. Use linearity of expectation to compute the expected number of fixpoints (indices that contain the same number before and after).

   Clearly describe the sample space and the random variable that you are using, and break your overall random variable into multiple (indicator) random variables.

3. **Best case for quicksort**
   Let “Deterministic Quicksort” be the non-randomized Quicksort which takes the first element as a pivot, using the partition routine that we covered in class on slide 12 of the randomized algorithms slides.

   In the best case the pivot always splits the array in half, for all recursive calls of Deterministic Quicksort. Give a sequence of 3 distinct numbers and a sequence of 7 distinct numbers that cause this best-case behavior. *(Hint: For the sequence of 7 numbers the first two recursive calls should be on sequences of 3 numbers each.)*