CMPS 2200 Introduction to Algorithms – Fall 17

11/8/17

9. Homework

Due **11/15/17** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Bellman-Ford (8 points)**
   Let $G = (V, E)$ be a weighted, directed graph that possibly has negative weights. Let $|V| = n$ and $|E| = m$, and let $s \in V$ be a source vertex.

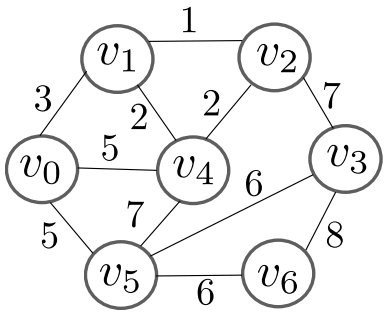
   Bellman-Ford's algorithm takes $O(nm)$ time, because it makes $n - 1$ passes relaxing all edges. In practice, this large number of passes may not always be necessary.

   (a) (7 points) Suggest a simple change to the Bellman-Ford algorithm such that the algorithm does not hard-code $n - 1$ many passes, but that it makes only the number of passes necessary to guarantee that all d-values are correctly computed.

   Give your answer in pseudo code. Justify why your code correctly computes the d-values.

   (b) (1 points) How can you express the runtime of your algorithm? It should be different from $O(nm)$.
   (Hint: What if you are allowed to introduce another variable, other than m and n, to express the runtime?)

2. **MST (12 points)**



   (a) (2 points) What is the cost of a minimum spanning tree in the graph above?

   (b) (4 points) How many minium spanning trees does this graph have? List them all.

   (c) (6 points) Run Prim's algorithm on this graph. Similar to question 1 on homework 8 for Dijkstra's algorithm, list all the different stages: The key for each vertex, the priority queue, the vertex extracted from the priority queue, and the predecessor array storing the tree.