CMPS 2200 Introduction to Algorithms – Fall 17

9/6/17

2. Homework

Due **9/13/17** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Big-Oh ranking (8 points)**
   Rank the following nine functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

   $$4^n \ , \ 3^n \ , \ \sqrt[2]{n} \ , \ 2n^3 + 4 \ , \ 1 \ , \ \log n \ , \ \log\log n \ , \ \sqrt[3]{n} \ , \ \sqrt[2]{n^6}$$

   Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

   $$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

   if the limits exist; where f' and g' are the derivatives of f and g, respectively.

2. **Transitivity (4 points)**
   Use the definition of big-Oh to prove:

   $$\text{If } f(n) \in O(g(n)) \text{ and } g(n) \in O(h(n)) \text{ then } f(n) \in O(h(n))$$

3. **Code snippet (4 points)**

   Give the Θ-runtime for the code snippet below, depending on n. Justify your answer.

```
for(i=n*n; i>=1; i=i-4)
   for(j=n*n; j>=1; j=j/4)
      print(" ");
```