CMPS 2200 Introduction to Algorithms – Fall 17

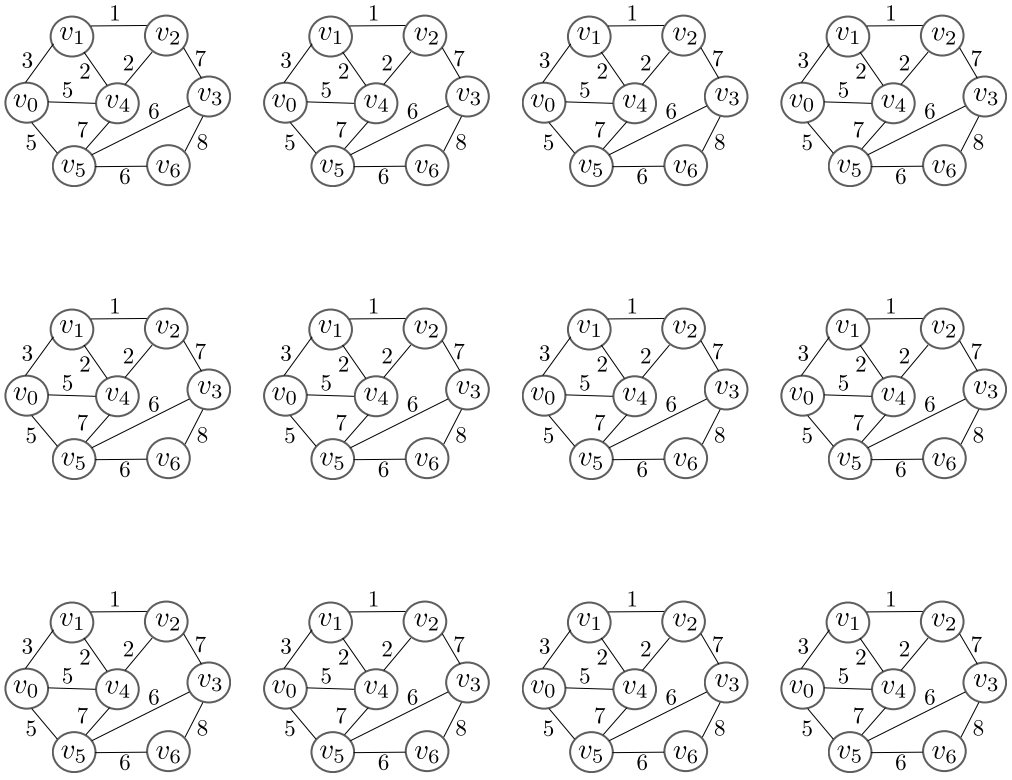
11/15/17

10. Homework

Due **11/29/17** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Kruskal (8 points)**
   Run Kruskal's algorithm on the graph below. Show all the different stages of the algorithm (tree edges and the set of vertex subsets). Clearly indicate the minimum spanning tree.



2. **Binary Counter (5 points)**
   Use aggregate analysis to show that, over a sequence of n increment operations on a k-bit binary counter, the amortized runtime of one such increment operation is $O(1)$.
   (Hint: Study the flipping behavior of every single bit $A[i]$.)

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3. **Queue from Stacks (7 points)**

Assume we are given an implementation of a stack, in which PUSH and POP operations take constant time each. We now implement a FIFO queue using two stacks A and B as follows:

ENQUEUE(x):
- Push x onto stack A

DEQUEUE():
- If stack B is nonempty, return B.POP()
- Else, pop all elements from A and immediately push them onto B. Return B.POP()

(a) (2 points) What is the worst-case runtime of a single ENQUEUE operation? What is the worst-case runtime of a single DEQUEUE operation?

(b) (5 points) Prove using the accounting method that the amortized runtime of ENQUEUE and DEQUEUE each is $O(1)$. Argue why your account balance is always non-negative.