

1. Homework

Due **9/6/17** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. Piazza (2 points)

Sign up for Piazza.

2. Bubblesort (10 points)

Bubblesort is a sorting algorithm that works by repeatedly swapping adjacent elements that are out of order. Consider the version of Bubblesort below which sorts the array $A[1..n]$ into increasing order by repeatedly “bubbling the minimum element of $A[i..n]$ to the left”.

```
Bubblesort(A[1..n]){
  for(i=1; i<=n; i++){
    //Loop invariant here

    //Bubble min of A[i..n] to the left, and store it in A[i]
    for(j=n; j>=i+1; j--){
      if(A[j]<A[j-1])
        swap A[j] with A[j-1]
    }
  }
}
```

Let $A'[1..n]$ denote the output of `Bubblesort(A)`. To prove that Bubblesort is correct, we need to prove that $A'[1] \leq A'[2] \leq \dots \leq A'[n]$.

- (2 points) State a loop invariant for the outer for-loop that will allow you to prove the correctness of the algorithm. (Assume that you know that the inner for-loop bubbles the minimum of $A[i..n]$ to the left.)
- (4 points) Use the loop invariant to prove the correctness of the algorithm. For this you need to prove by induction that the loop invariant holds for all iterations of your loop (“base step” and “inductive step”), and then use the loop invariant in the “termination step” to prove the correctness of the algorithm.
- (4 points) Give best-case and worst-case running times in asymptotic (i.e., big-Oh) notation. Justify your answers. Also give example inputs attaining these runtimes.

3. Big-Oh (12 points)

Use the definition of big-Oh:

- (3 points) Prove that $7n^5 + 2n^3 - 16 \in O(n^5)$.
- (3 points) Prove that $n^5 \in O(7n^5 + 2n^3 - 16)$.
- (3 points) Is $2^{n+1} \in O(2^n)$? Justify your answer.
- (3 points) Is $2^{2n} \in O(2^n)$? Justify your answer.