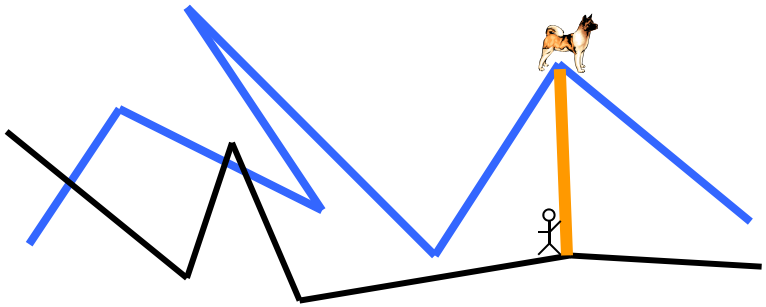
CMPS 2200 – Fall 2015

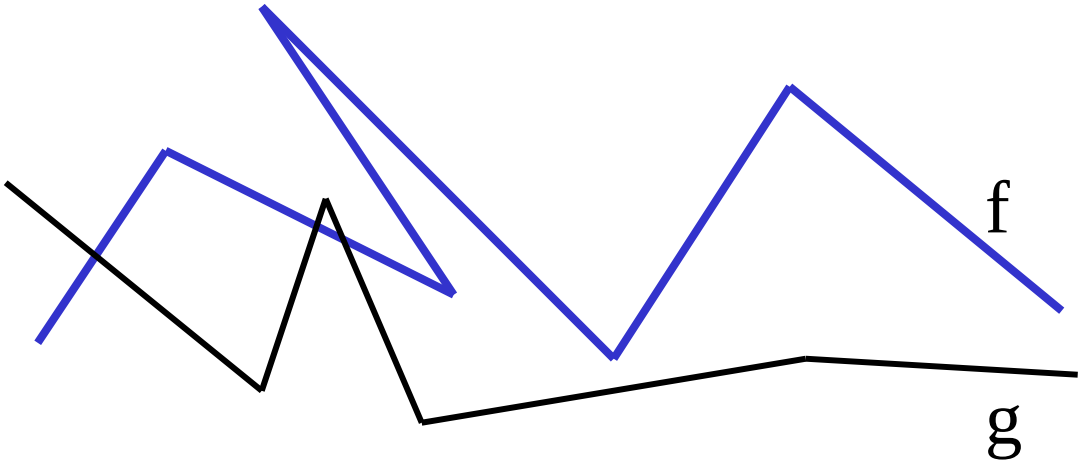


Fréchet Distance

Carola Wenk

Polygonal Curves

- Let f,g:[0,1] $\rightarrow R^d$ be two **polygonal curves** (i.e., piecewise linear curves)



- What are good distance measures for curves?
  - Hausdorff distance?
  - Fréchet distance?

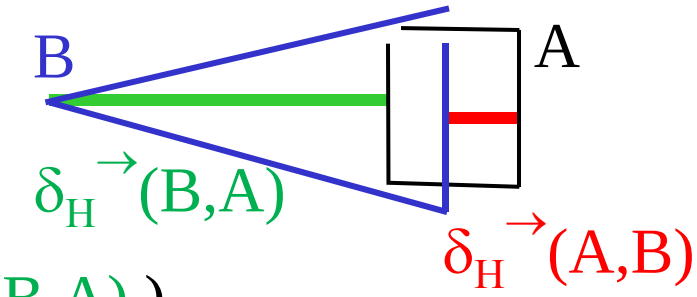
When Are Two Curves „Similar“?

- Directed Hausdorff distance

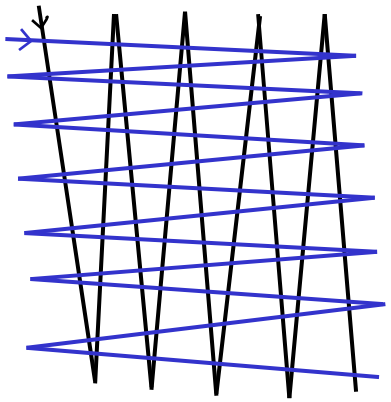
$$\delta_H^{\rightarrow}(A,B) = \max_{a \in A} \min_{b \in B} \| a-b \|$$

- Undirected Hausdorff-distance

$$\delta_H(A,B) = \max(\delta_H^{\rightarrow}(A,B), \delta_H^{\rightarrow}(B,A))$$



But:

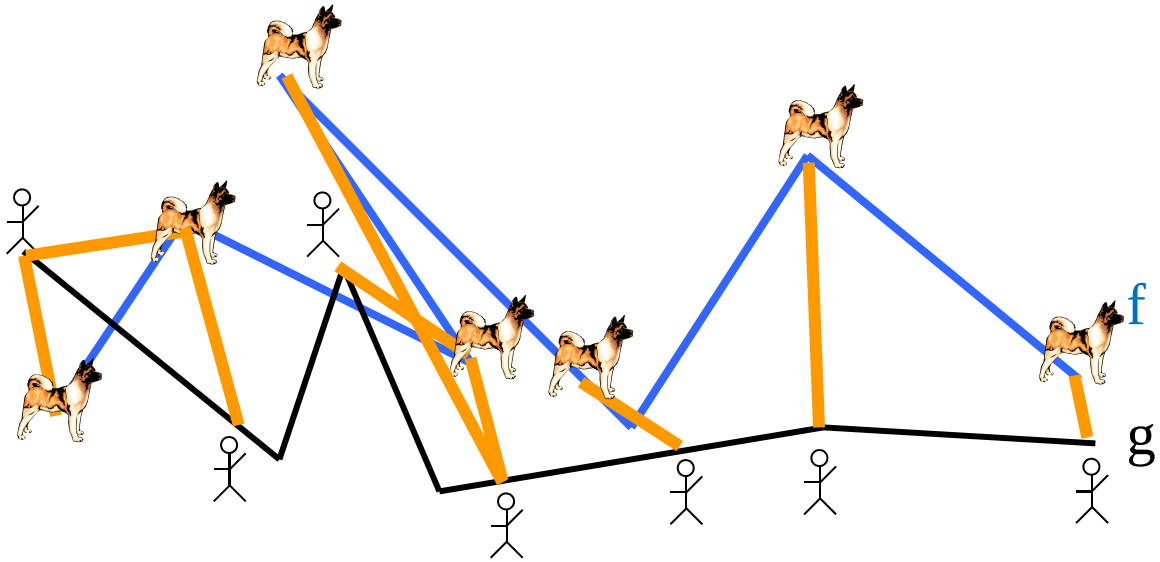


- Small Hausdorff distance
- When considered as curves the distance should be large
- The Fréchet distance takes the continuity of the curves into account

Fréchet Distance for Curves

$$\delta_F(f,g) = \inf_{\alpha,\beta:[0,1]\to[0,1]} \max_{t\in[0,1]} \|f(\alpha(t))-g(\beta(t))\|$$

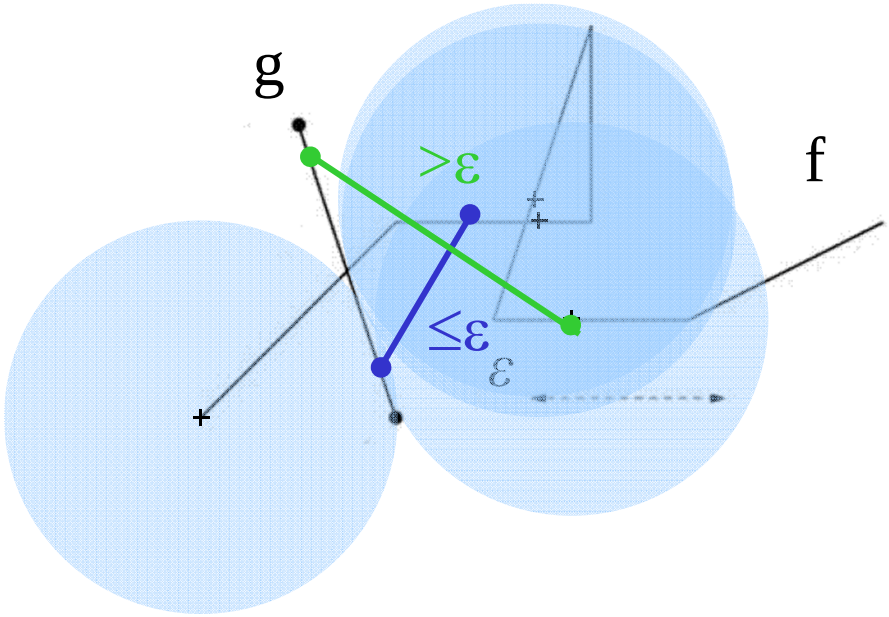
where α and β range over continuous monotone increasing reparameterizations only.

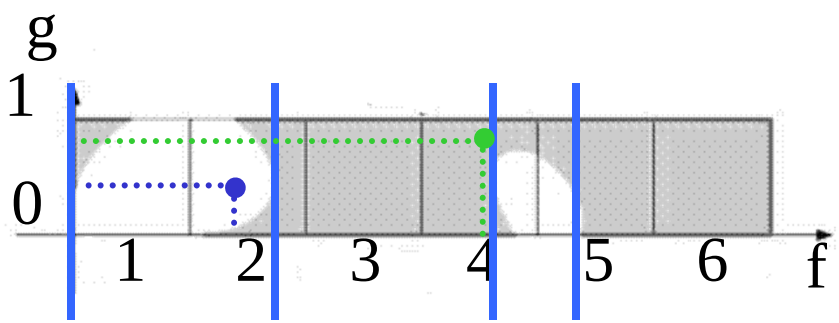


- Man and dog walk on one curve each
- They hold each other at a **leash**
- They are only allowed to go forward
- δ_F is the minimal possible leash length

[F06] M. Fréchet, Sur quelques points de calcul fonctionel, *Rendiconti del Circolo Mathematico di Palermo* 22: 1-74, 1906.

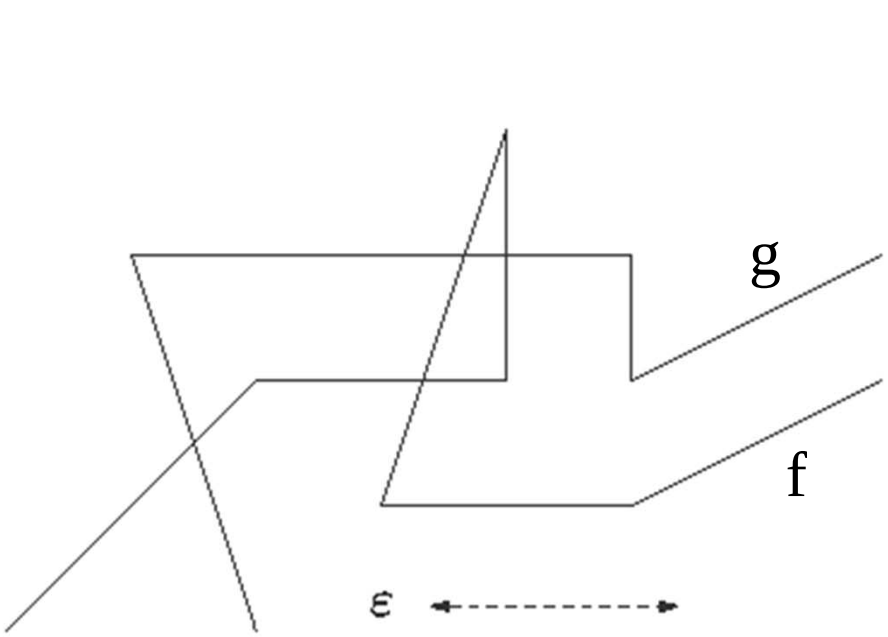
Free Space Diagram

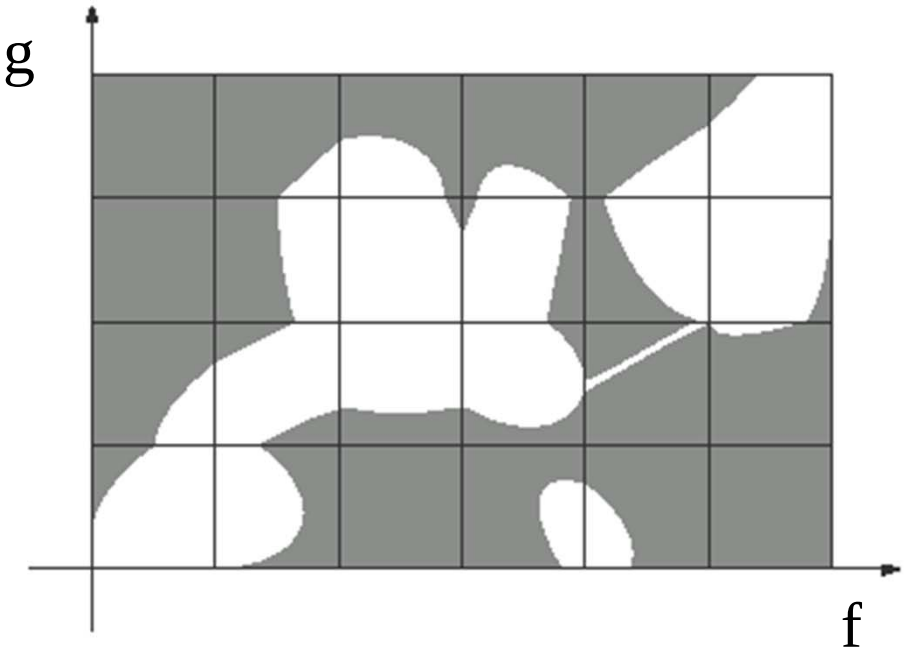




- Let ε > 0 fixed (eventually solve decision problem)
- $F_\varepsilon(f,g) = \{ (s,t) \in [0,1]^2 \mid \| f(s) - g(t) \| \leq \varepsilon \}$ *white points* **free space** of f and g
- The free space in one cell is an ellipse.

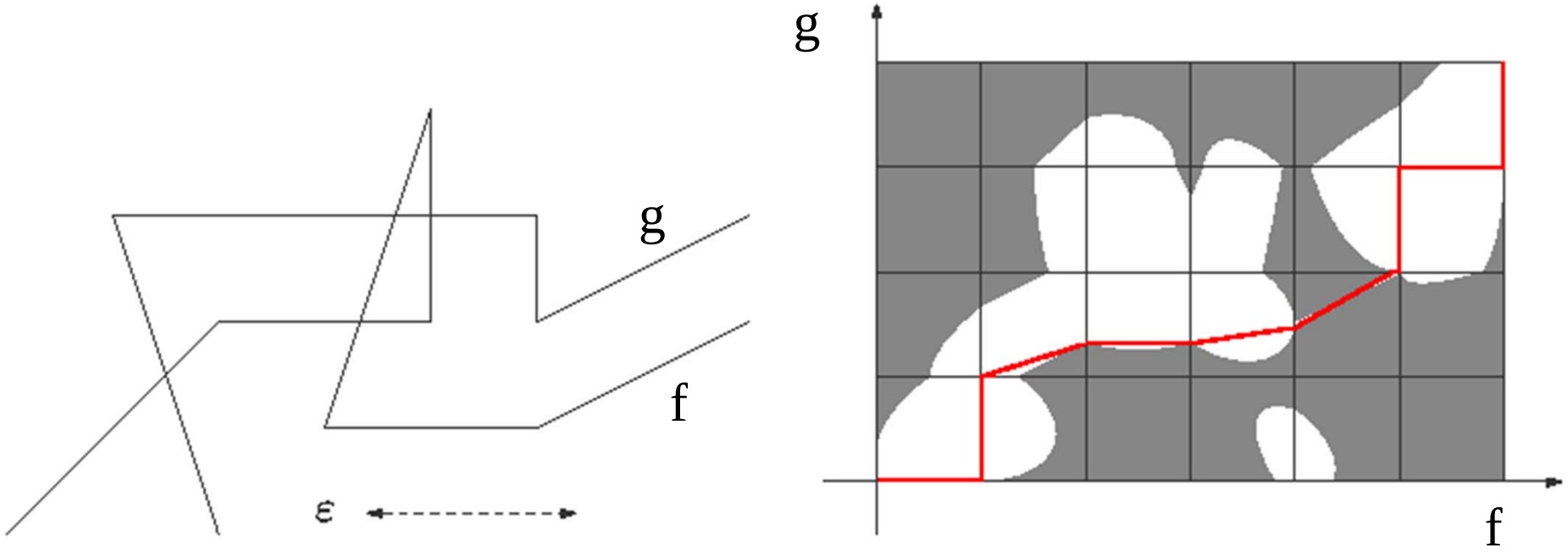
Free Space Diagram





- Let $\varepsilon > 0$ fixed (eventually solve decision problem)
- $F_\varepsilon(f,g) = \{ (s,t) \in [0,1]^2 \mid \| f(s) - g(t) \| \leq \varepsilon \}$ *white points* **free space** of f and g
- The free space in one cell is an ellipse.

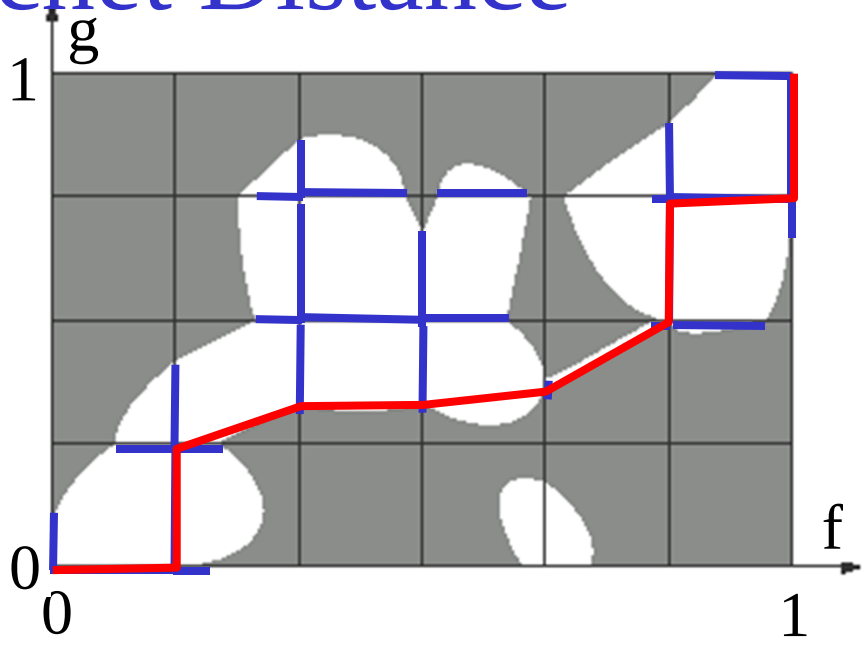
Free Space Diagram



- Monotone path encodes reparametrizations of f and g
- $\delta_F(f,g) \leq \varepsilon$ iff there is a monotone path in the free space from (0,0) to (1,1)

Compute the Fréchet Distance

- **Solve the decision problem**
  $\delta_F(f,g) \leq \varepsilon$ in O(mn) time:
  - Find monotone path using DP:
  - On each cell boundary compute the interval of all points that are reachable by a monotone path from (0,0)
  - Compute a monotone path by backtracking



- **Solve the optimization problem**
  - In practice in O(mn log b) time with binary search and b-bit precision
  - In O(mn log mn) time [AG95] using parametric search (using Cole's sorting trick)
  - In O(mn $\log^2$mn) expected time [CW09] with randomized red/blue intersections

[AG95] H. Alt, M. Godau, Computing the Fréchet distance between two polygonal curves, *IJCGA* 5: 75-91, 1995.
[CW10] A.F. Cook IV, **C. Wenk**, Geodesic Fréchet Distance Inside a Simple Polygon, ACM TALG 7(1), 19 pages, 2010.

GPS Trajectories for Dynamic Routing

- Navigation systems answer shortest path queries based on travel times on road segments

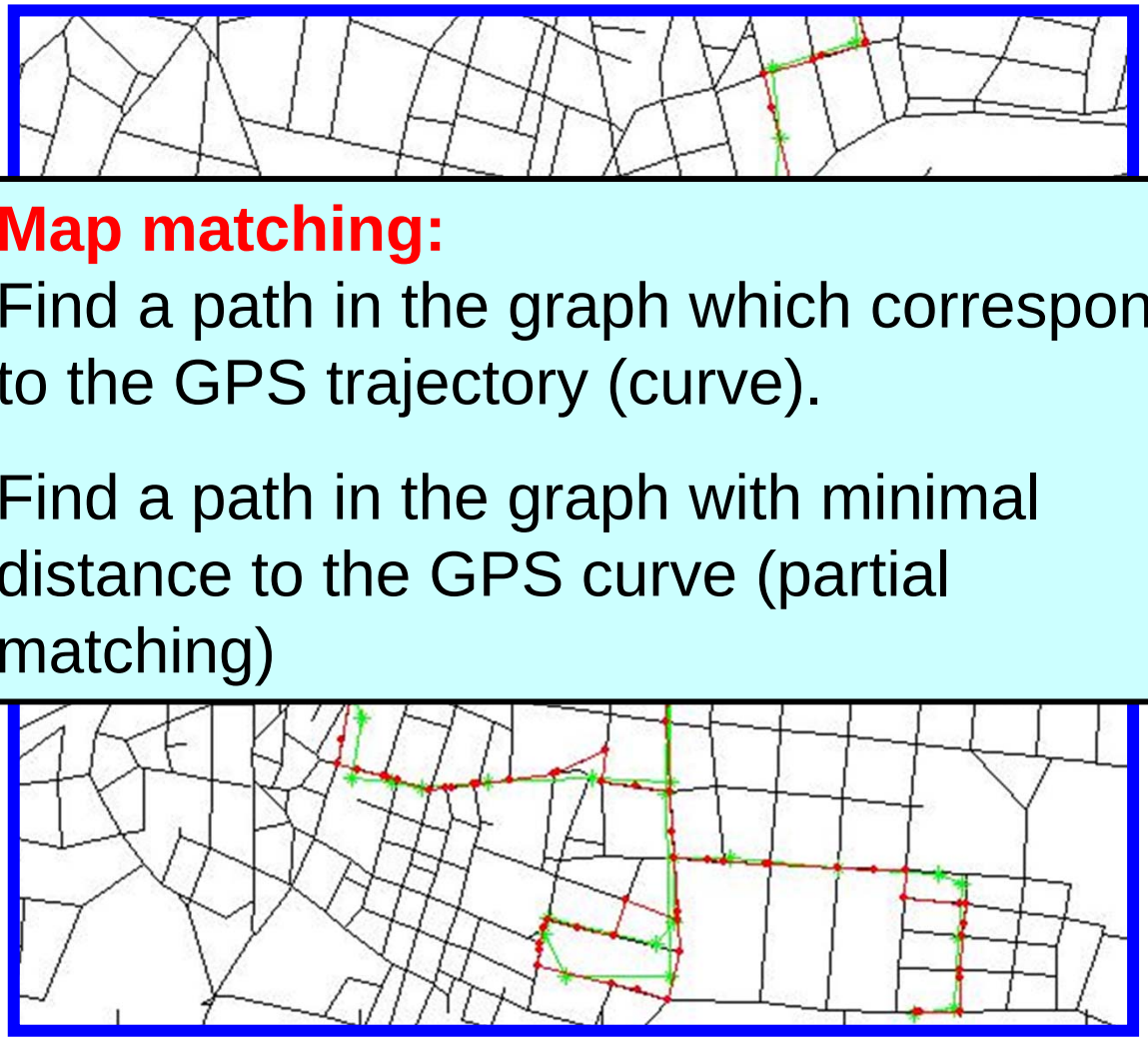
- How does one collect dynamic travel-time weights that are not just derived from speed limits?

- Use GPS trajectory data from large number of vehicles (vehicle fleets).

⇒Need to map trajectories to graph (road map)

⇒ Map-matching

GPS Trajectories from Vehicles



Map matching:
Find a path in the graph which corresponds to the GPS trajectory (curve).

Find a path in the graph with minimal distance to the GPS curve (partial matching)

1) Measurement error: GPS points generally do not lie on the road map

Sampling error: The GPS trajectory is a by-product and may be sampled just every 30s

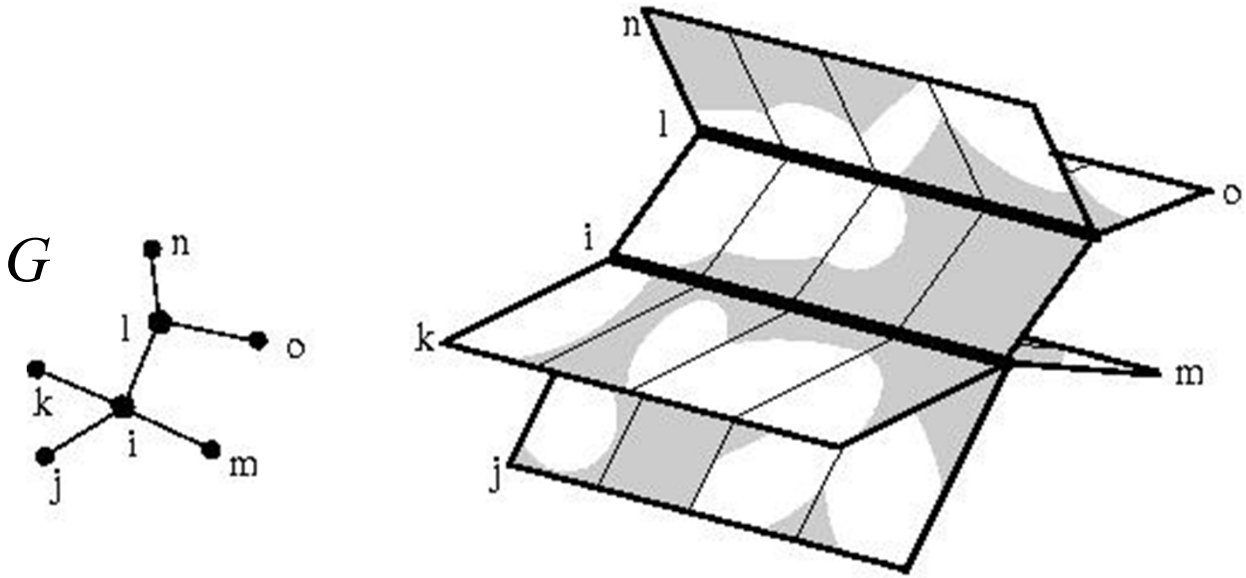
⇒ The GPS trajectory does not lie on the road map

Road map of Athens

GPS trajectoriy

Corresponding path in the road map

Free Space Surface

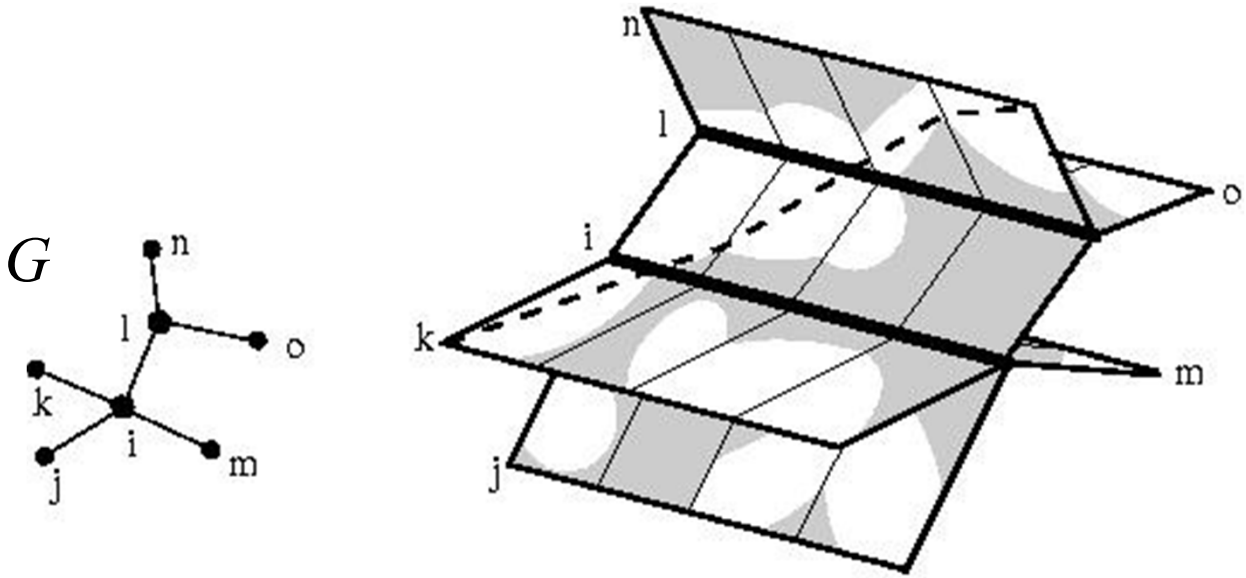


- Glue the free space diagrams $FD_{i,j}$ together according to adjacency information in *G*

→ **Free space surface of f and *G***

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

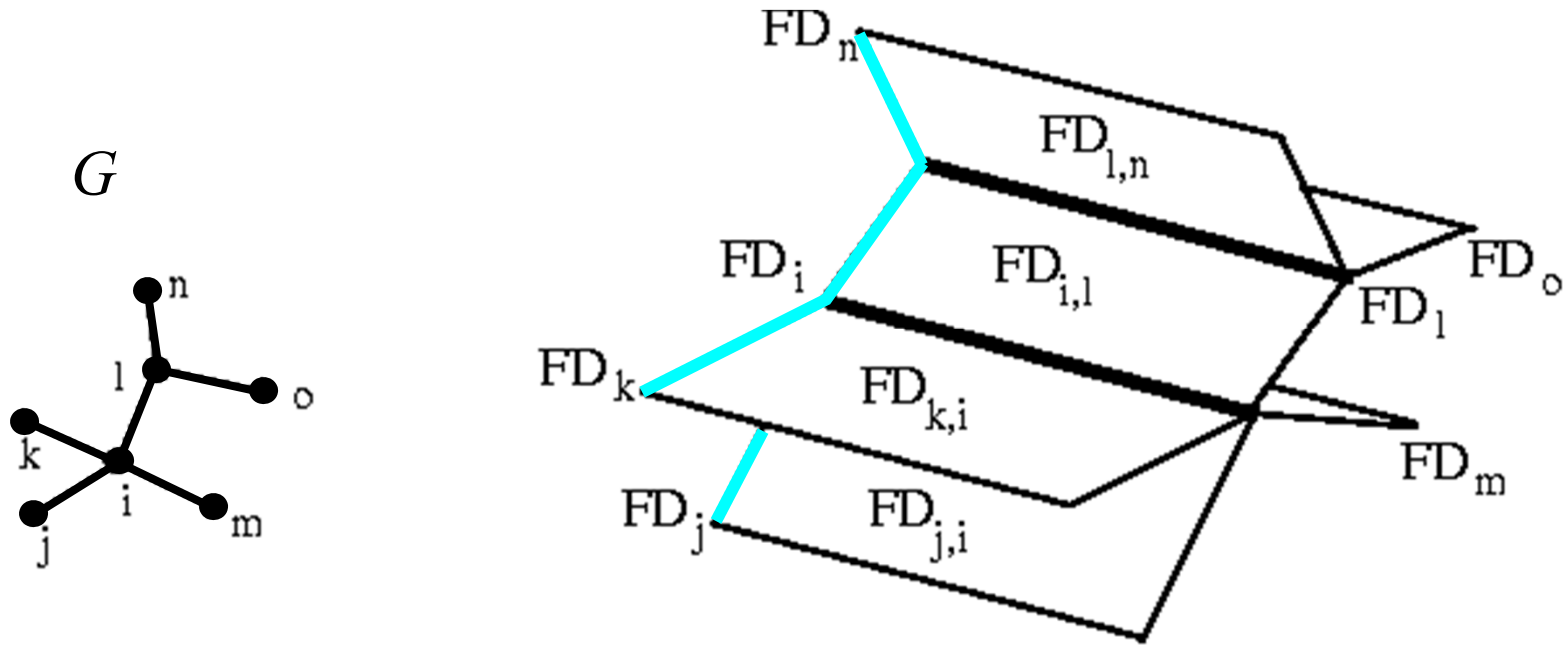
Free Space Surface



- **Task:** Find a **monotone path** in the free space surface that
  - starts in a lower left corner
  - and ends in an upper right corner

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

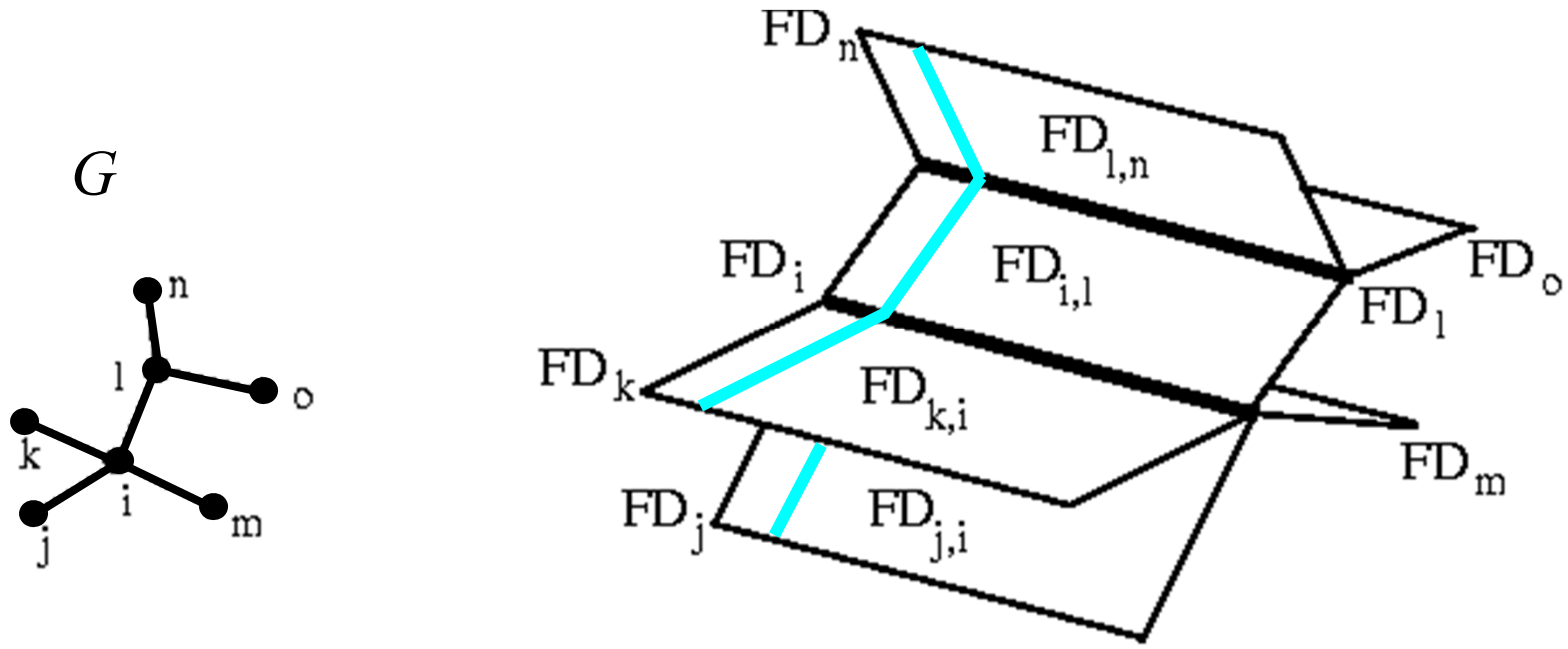
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

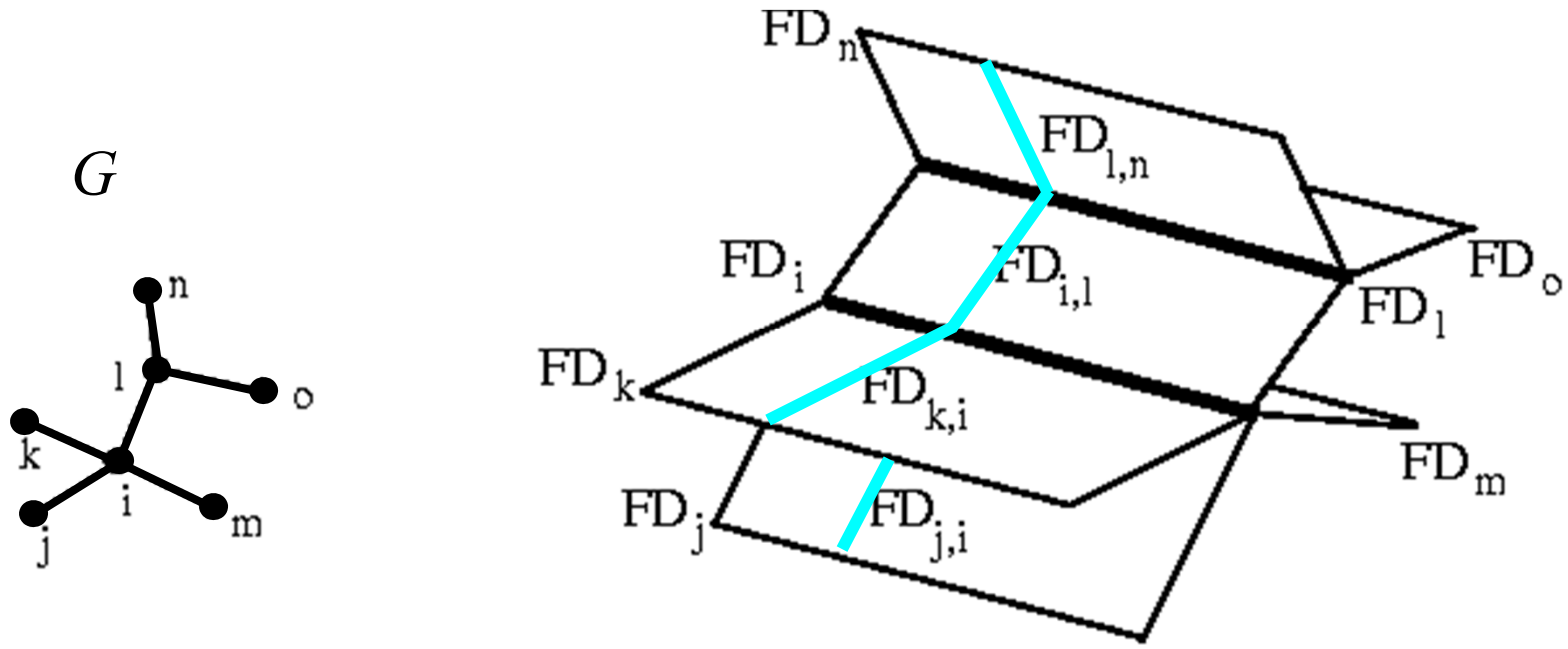
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

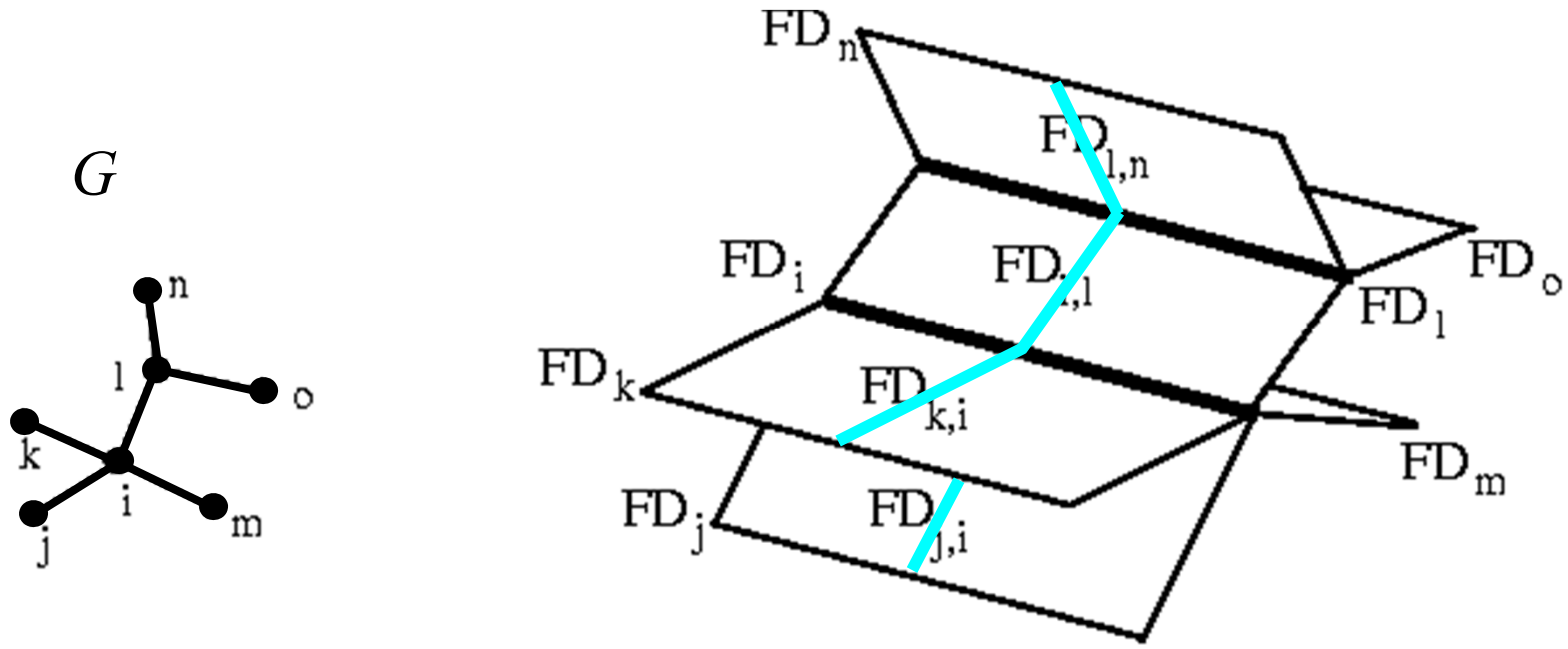
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

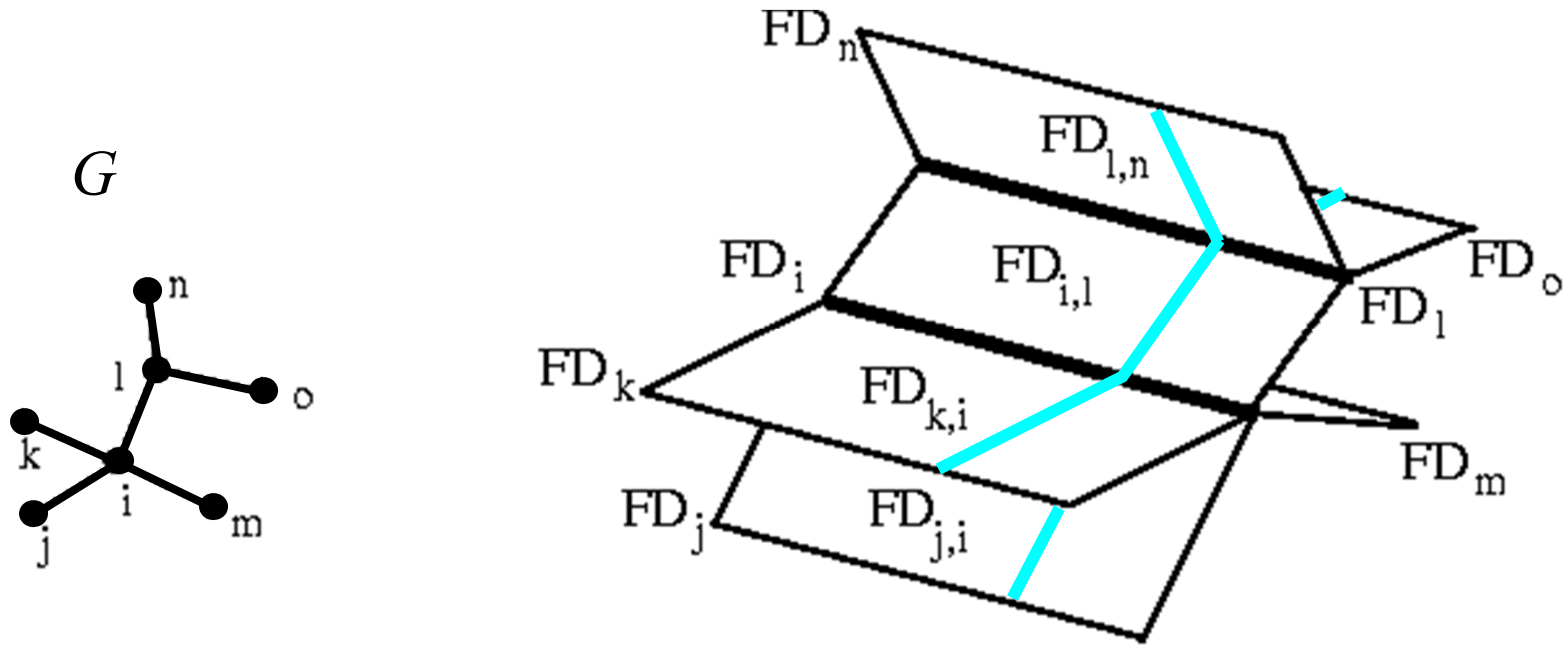
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

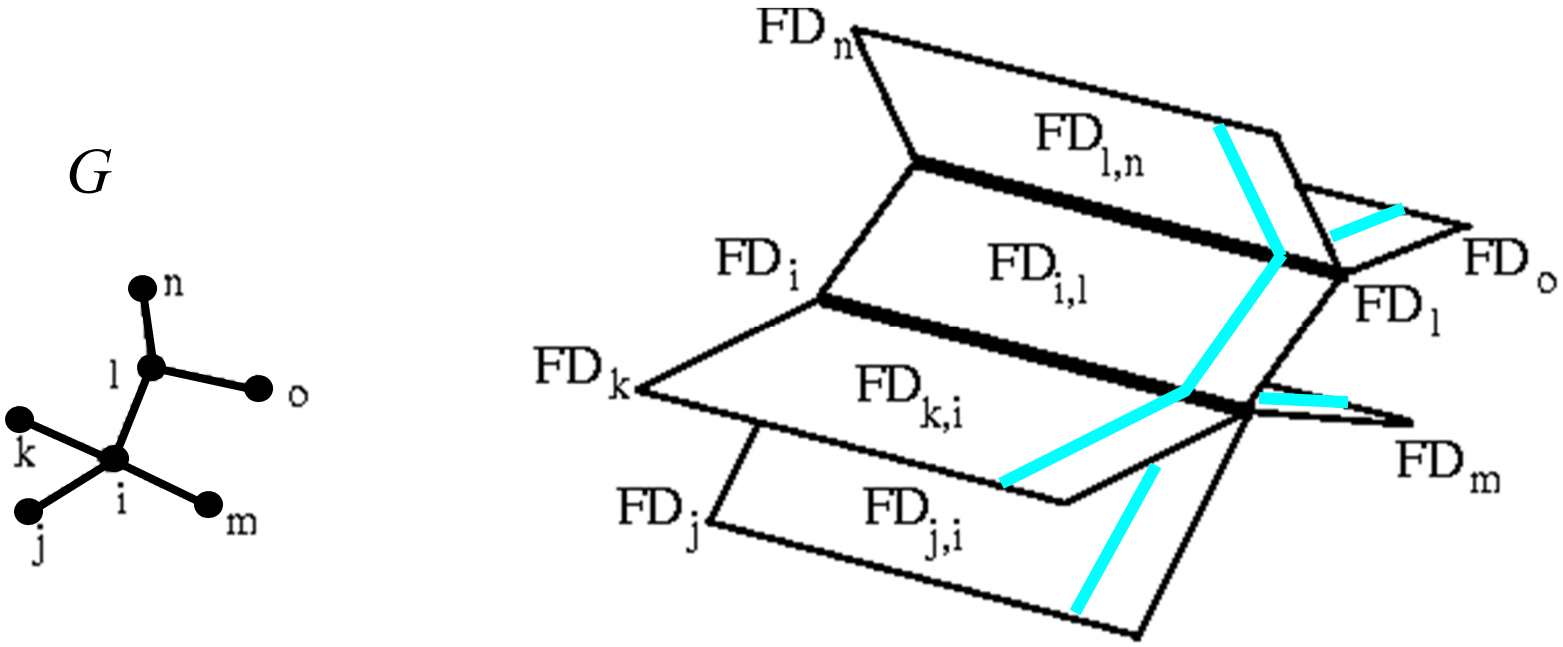
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

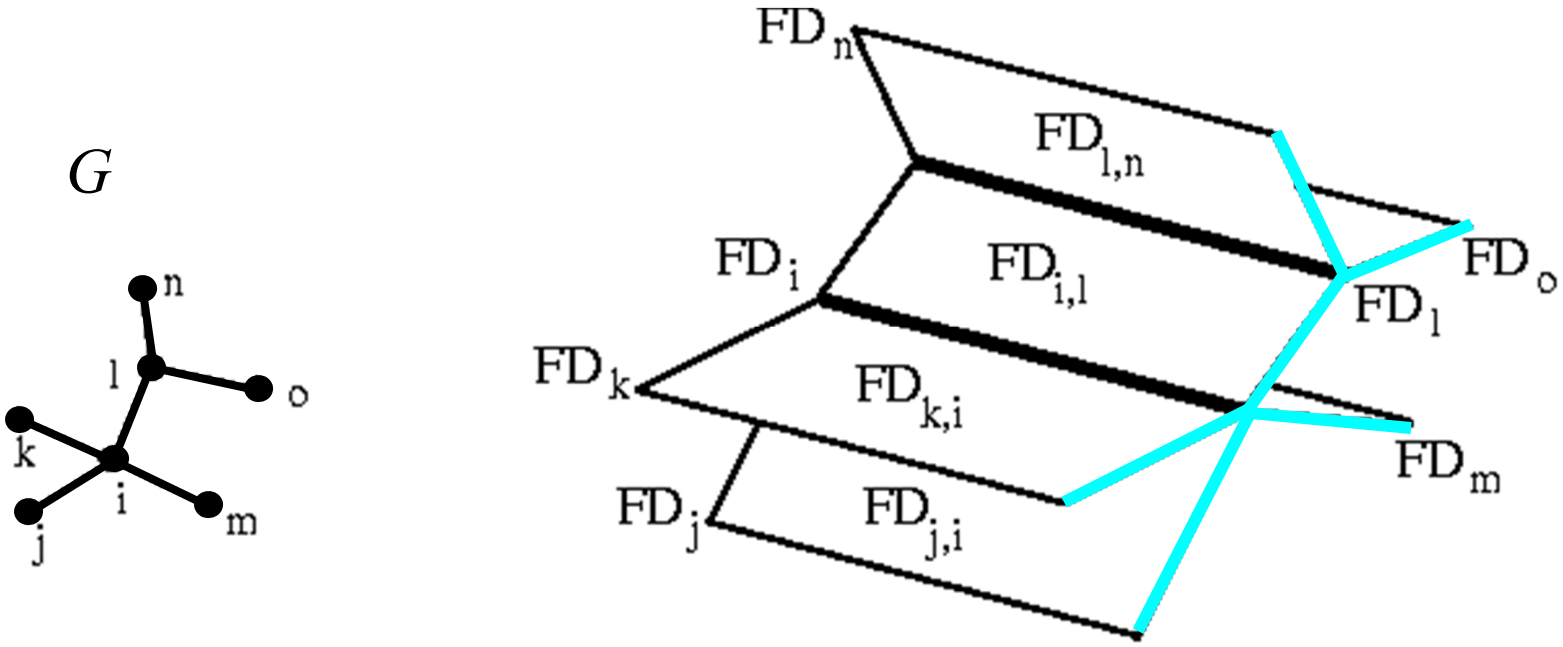
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

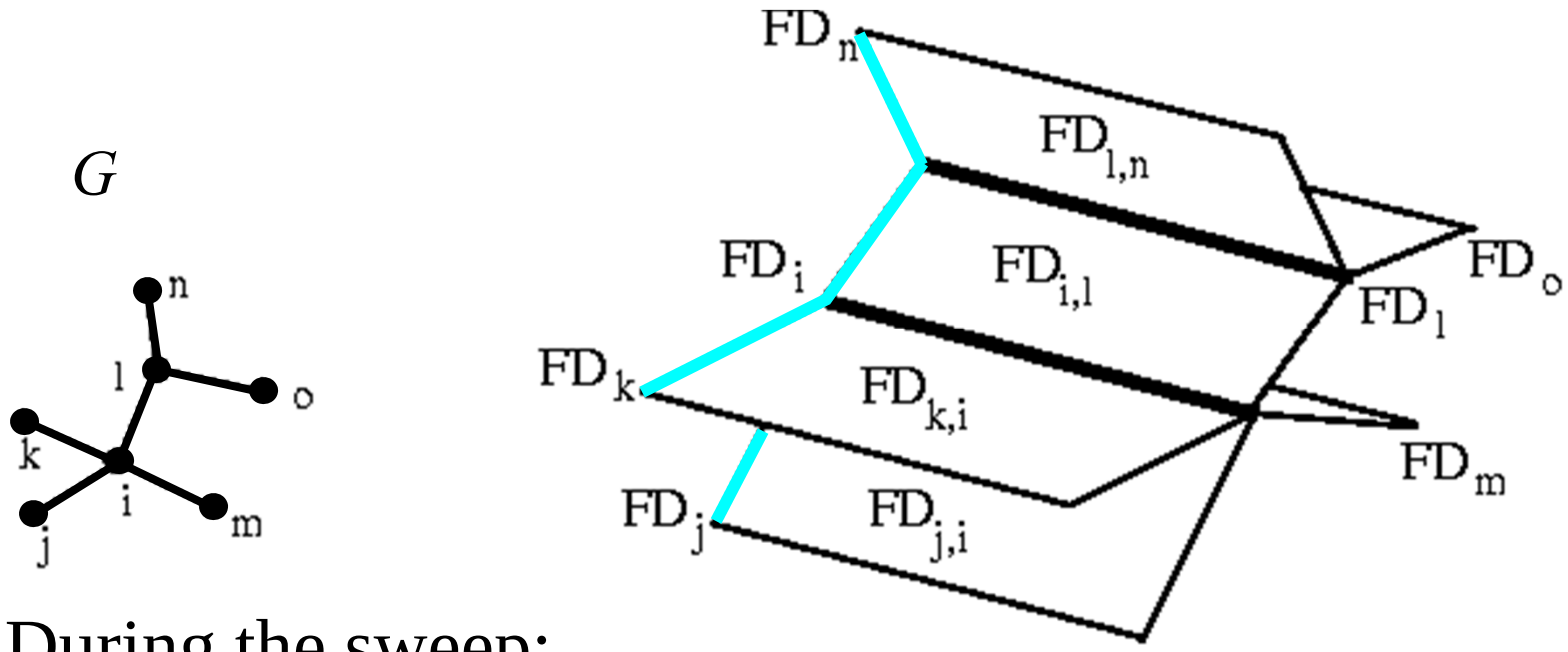
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Compute Reachable Points



- During the sweep:

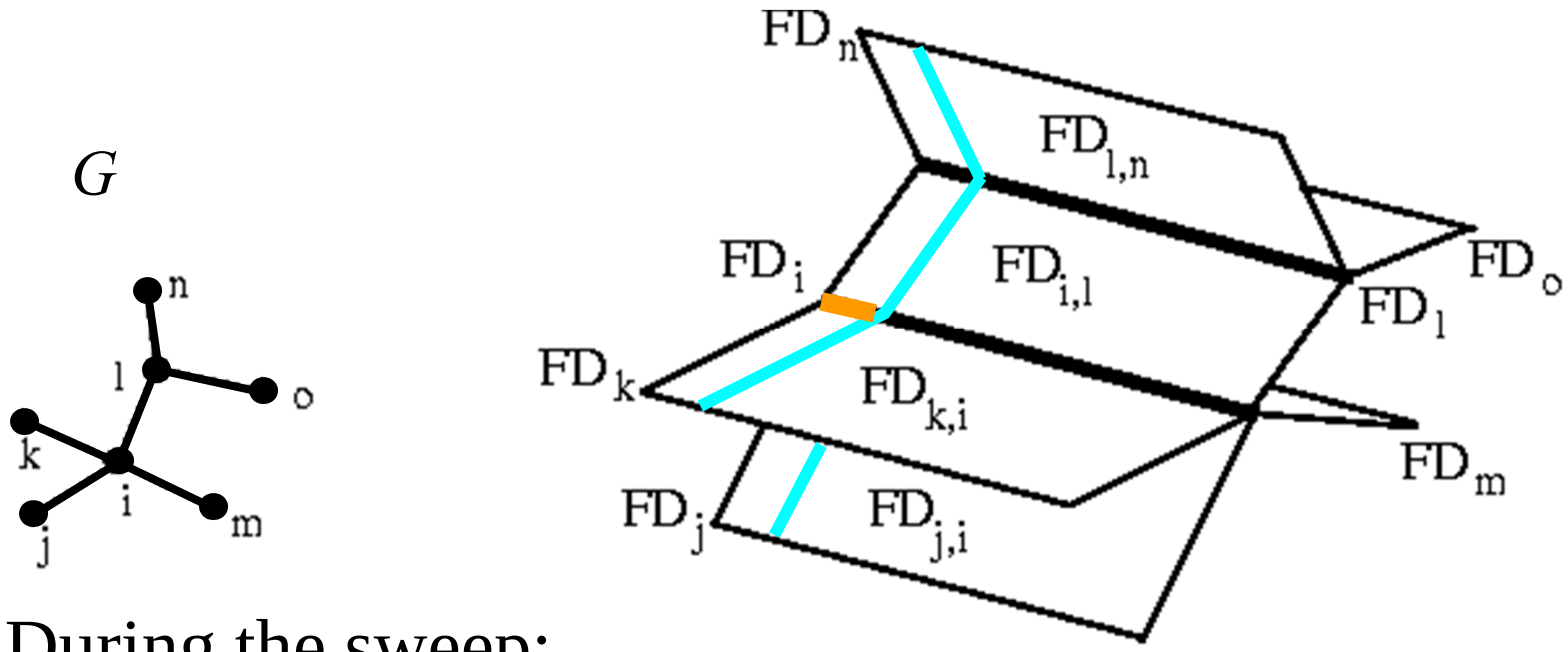
  Compute points on the free space surface, to the left of the sweep line, which are reachable by a monotone path from a lower left corner.

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Compute Reachable Points



- During the sweep:

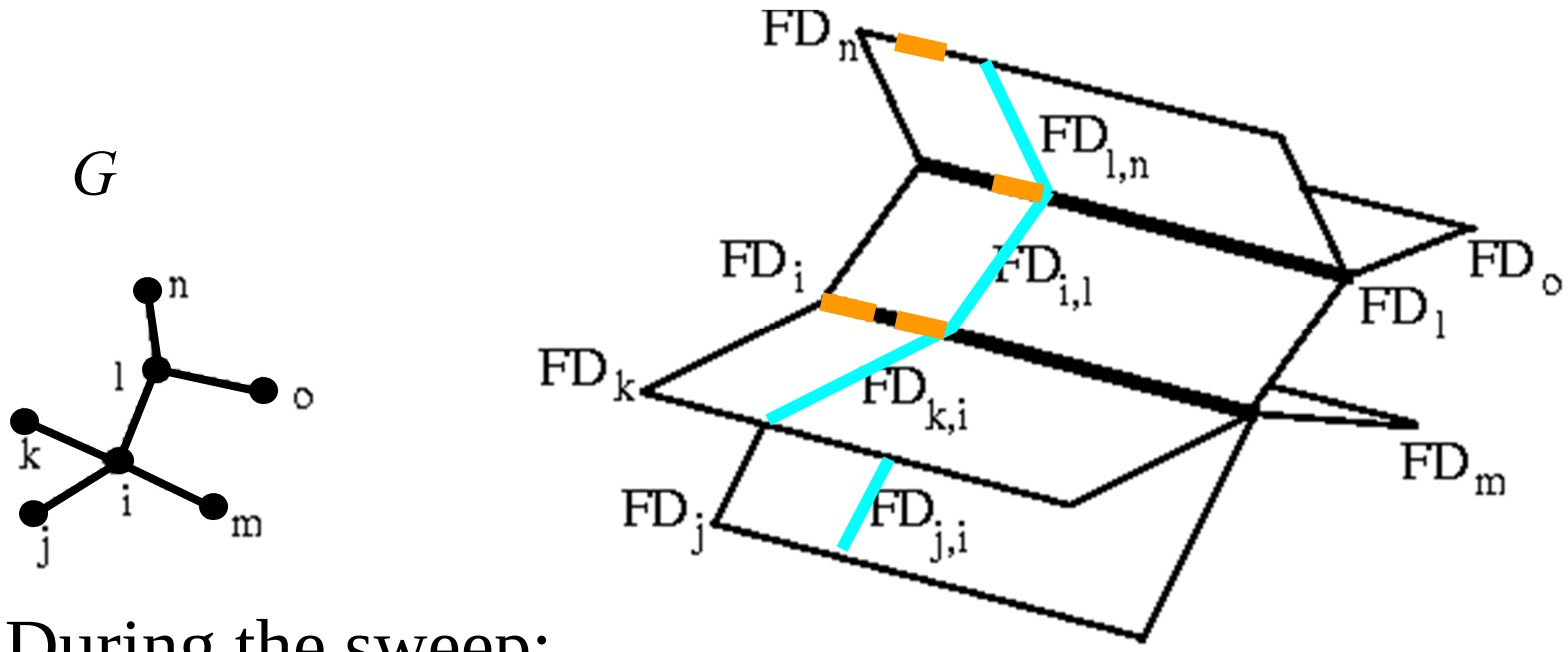
  Compute points on the free space surface, to the left of the sweep line, which are reachable by a monotone path from a lower left corner.

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Compute Reachable Points



- During the sweep:

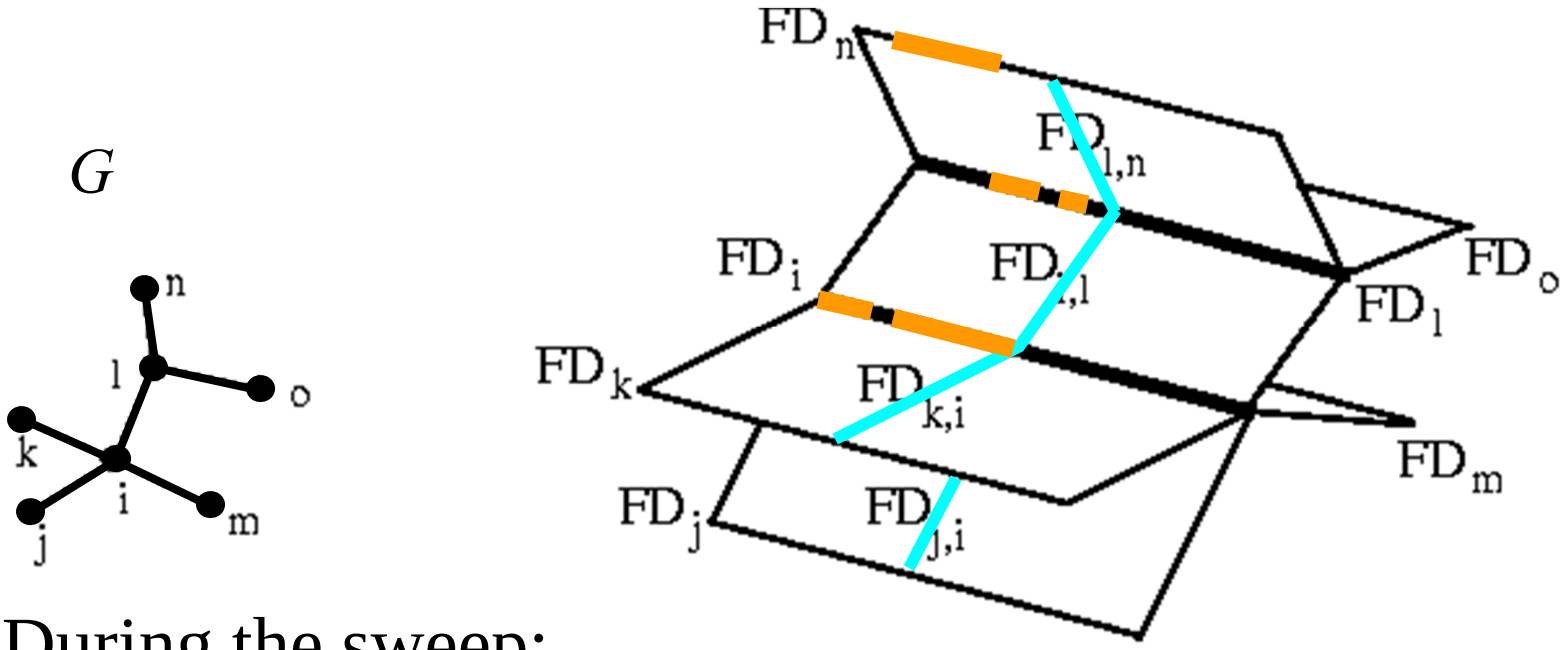
  Compute points on the free space surface, to the left of the sweep line, which are reachable by a monotone path from a lower left corner.

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Compute Reachable Points



- During the sweep:

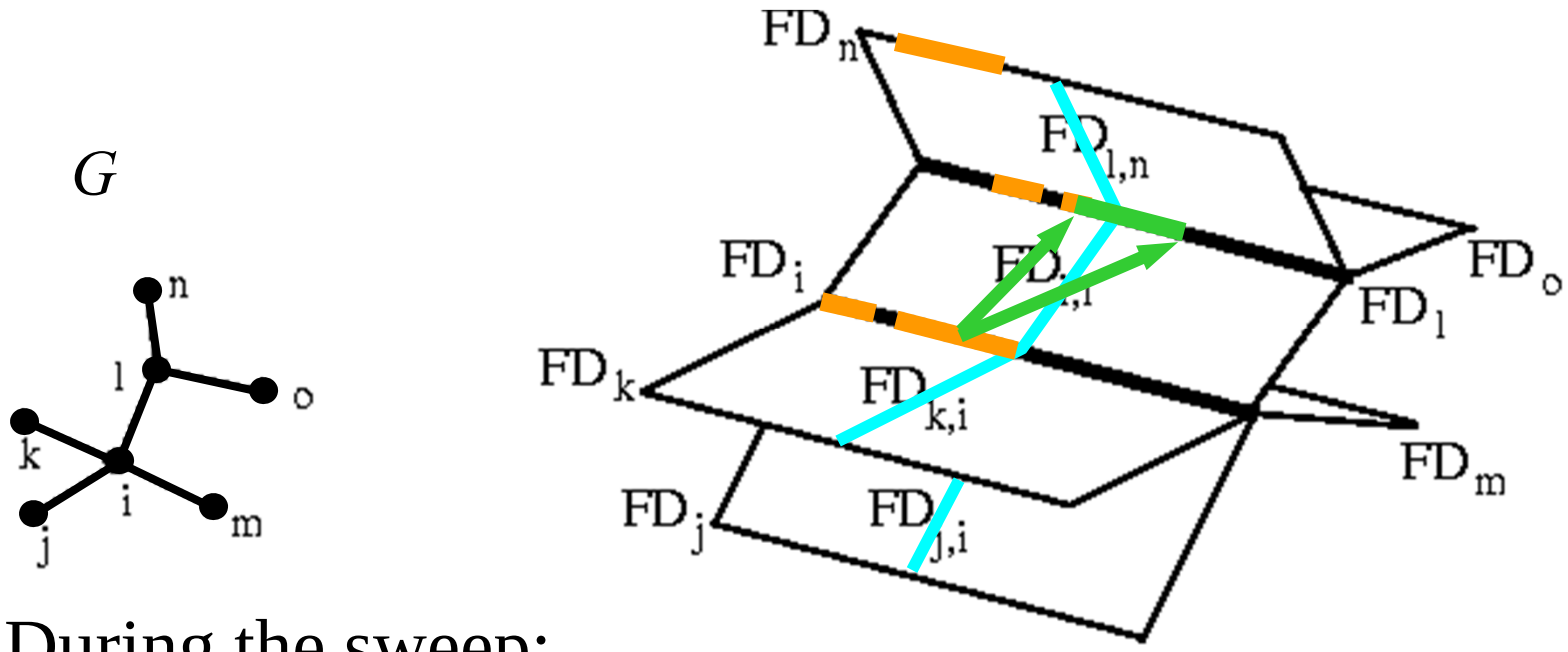
  Compute points on the free space surface, to the left of the sweep line, which are reachable by a monotone path from a lower left corner.

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

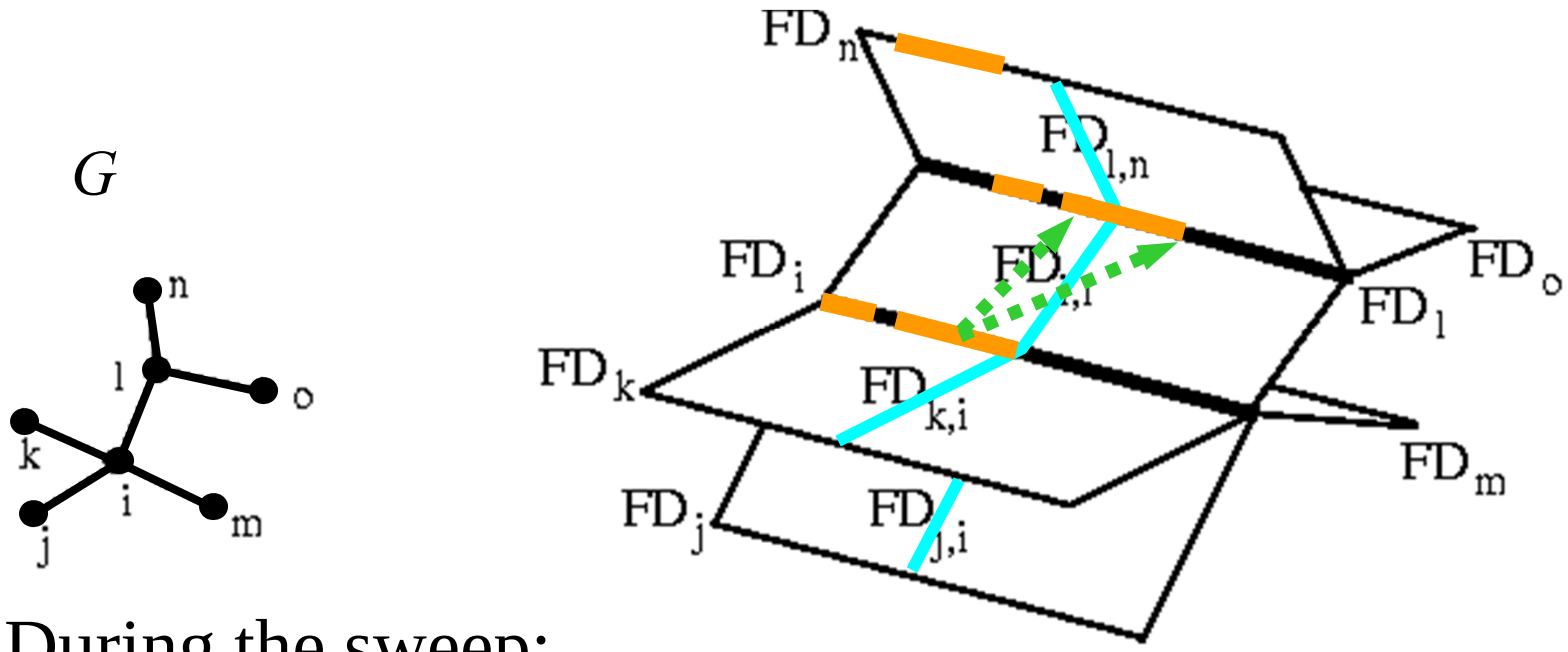
Update Reachable Points



- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

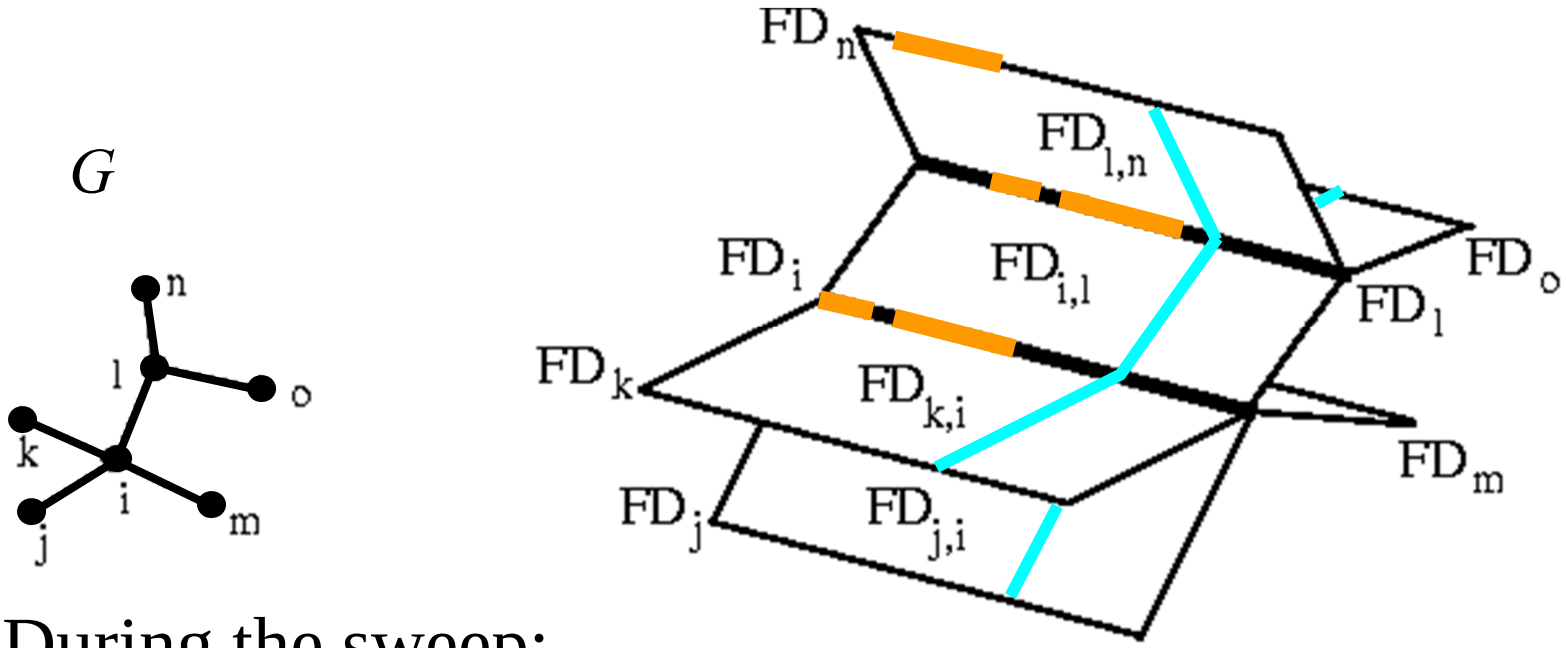
Update Reachable Points



- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Update Reachable Points



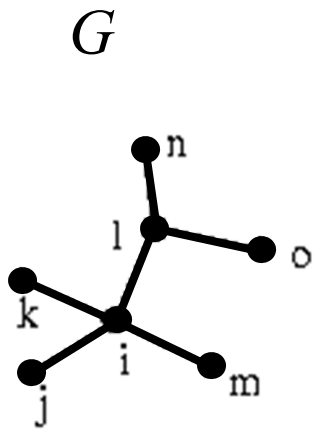
- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

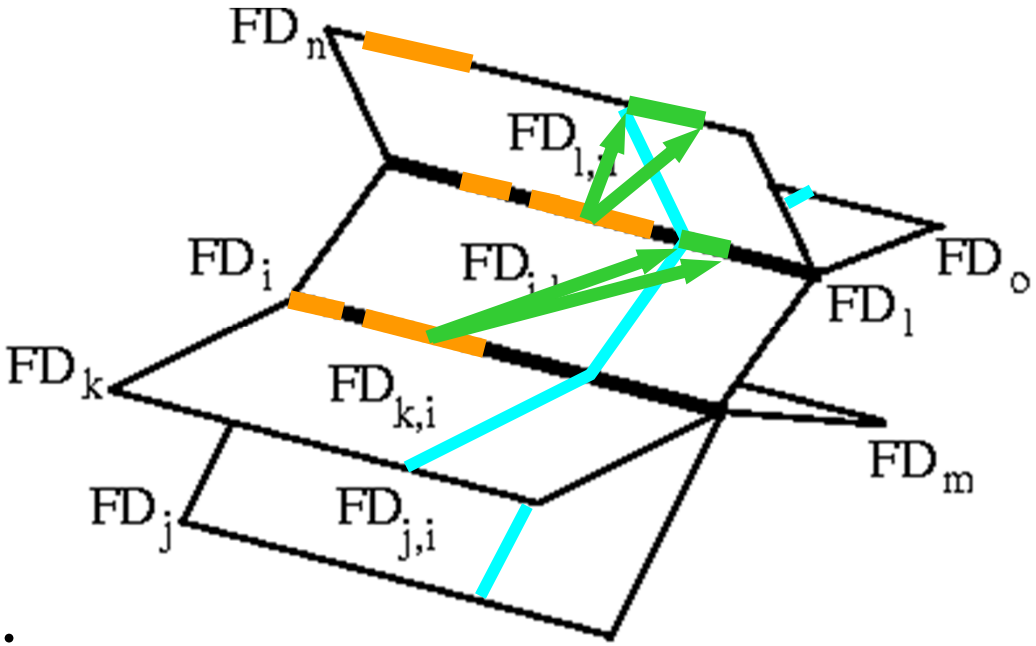
[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Update Reachable Points





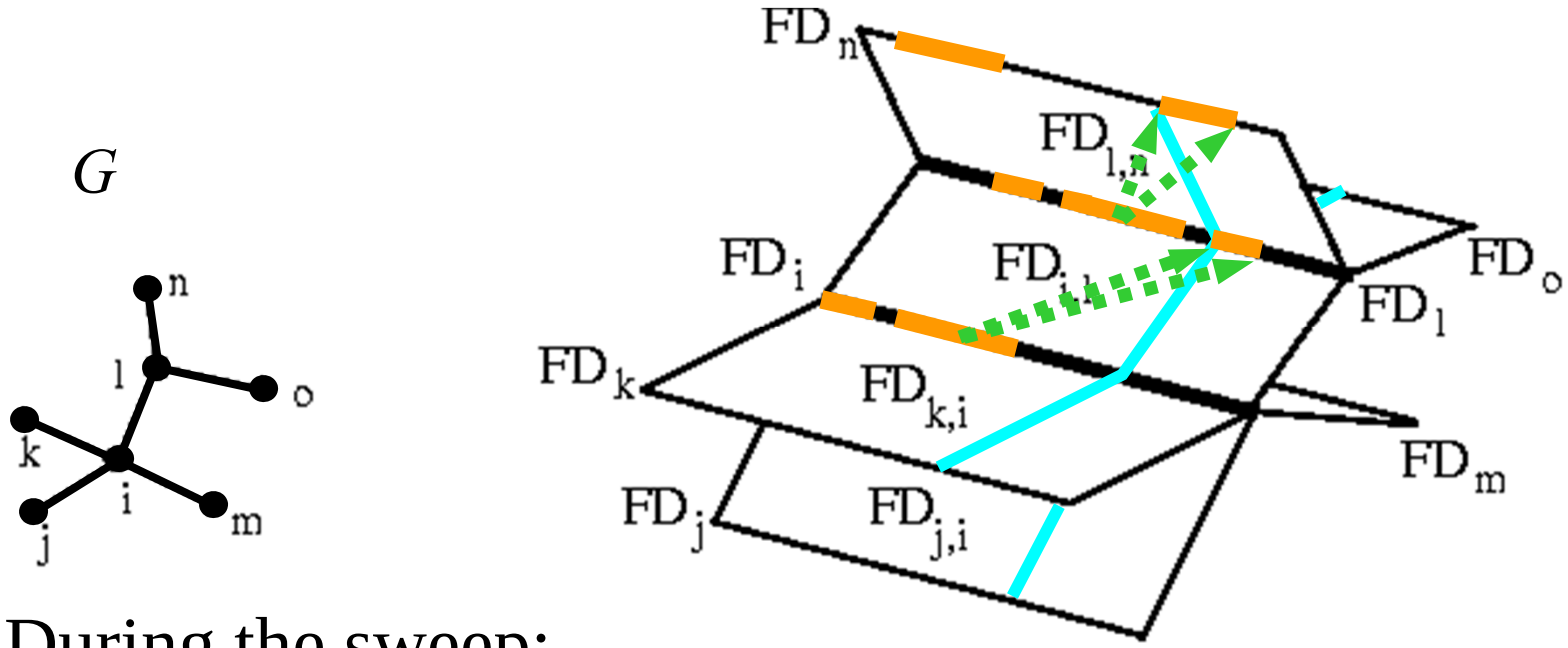
- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Update Reachable Points



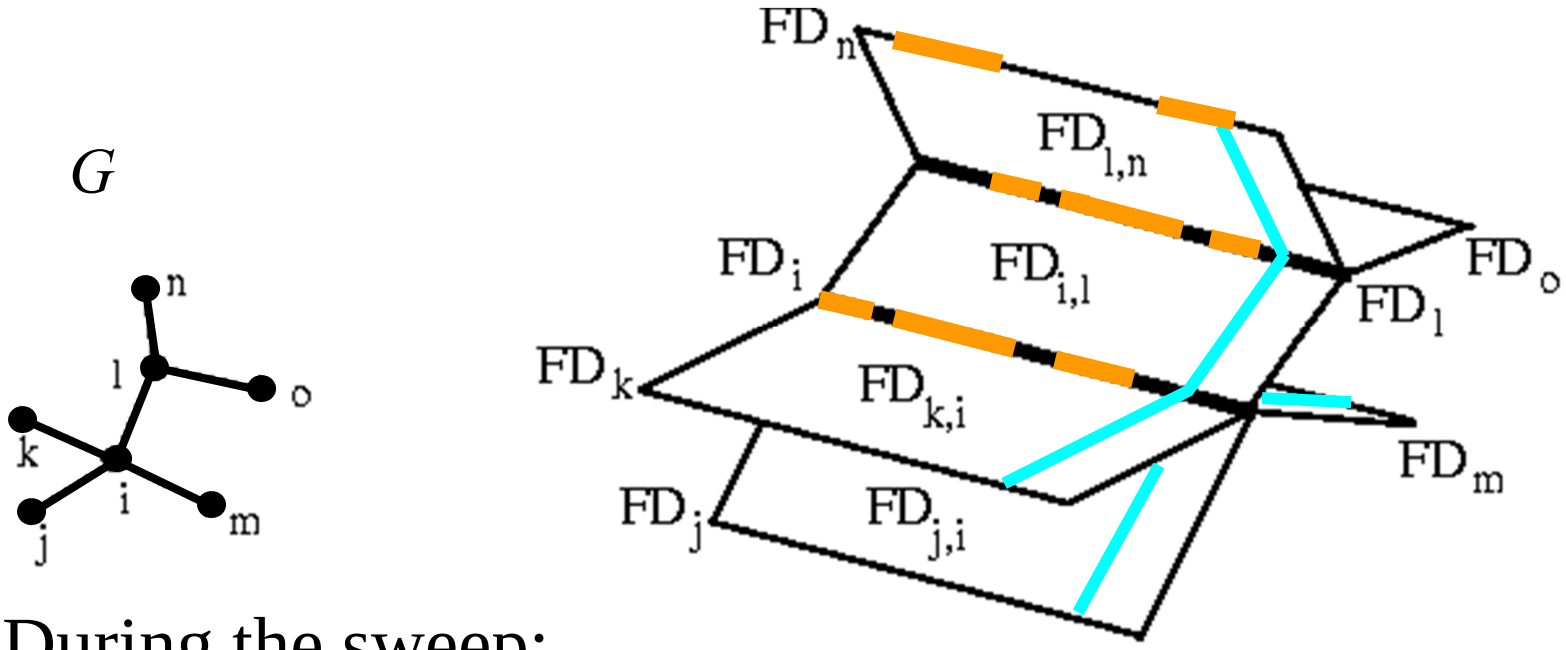
- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Update Reachable Points



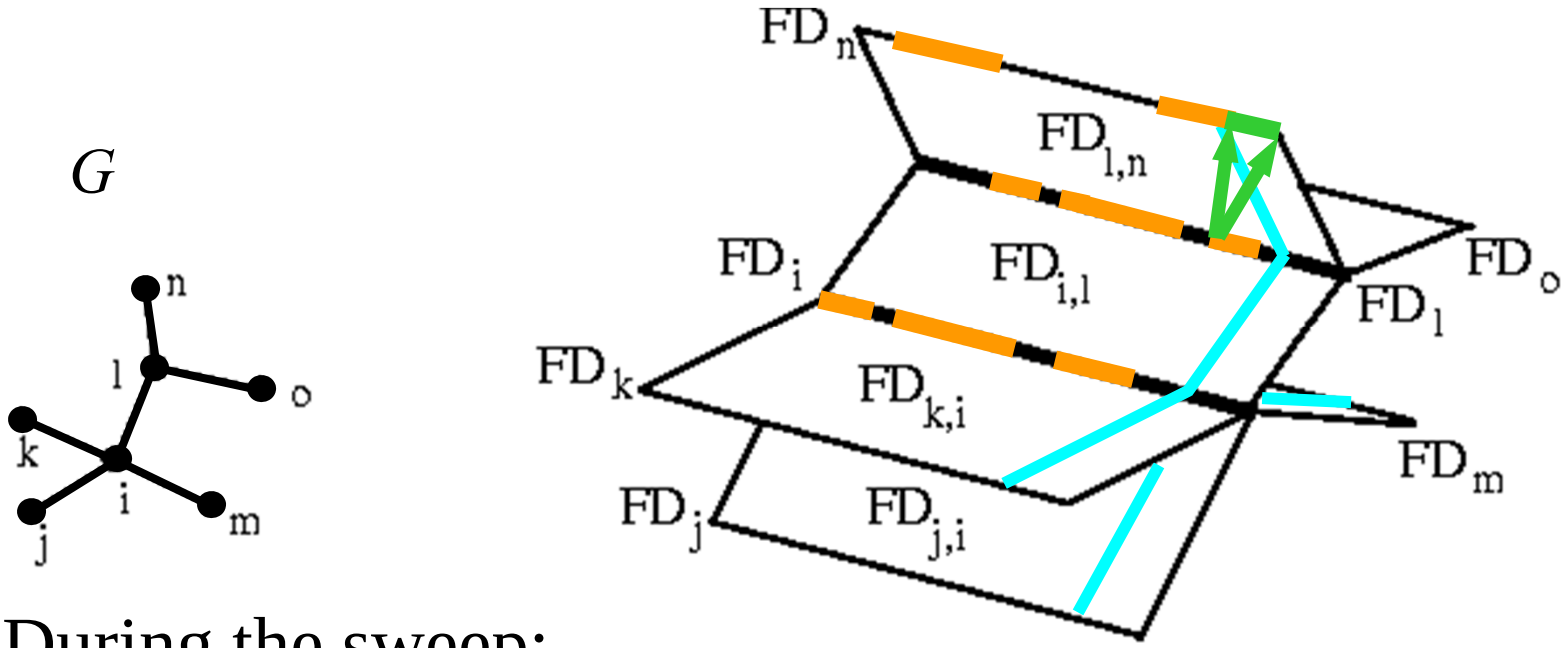
- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

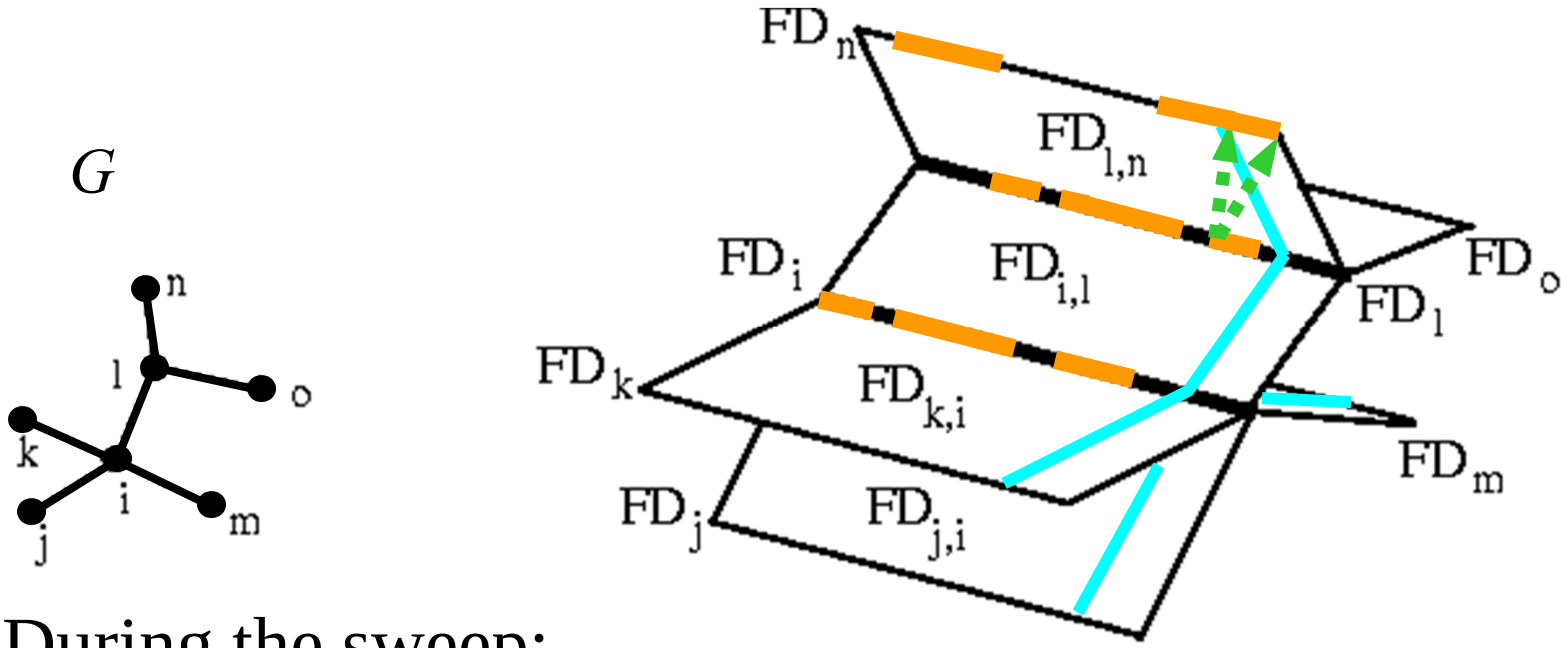
Update Reachable Points



- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Update Reachable Points



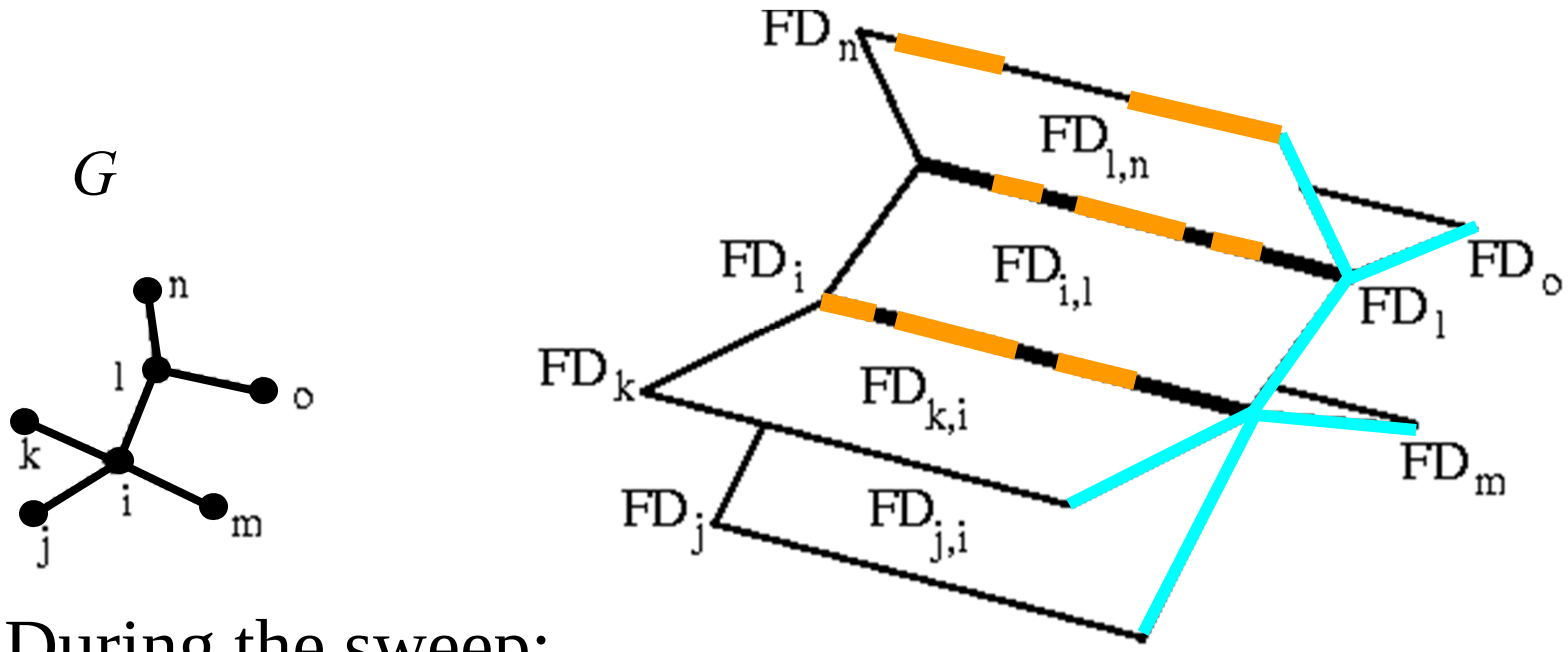
- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Update Reachable Points



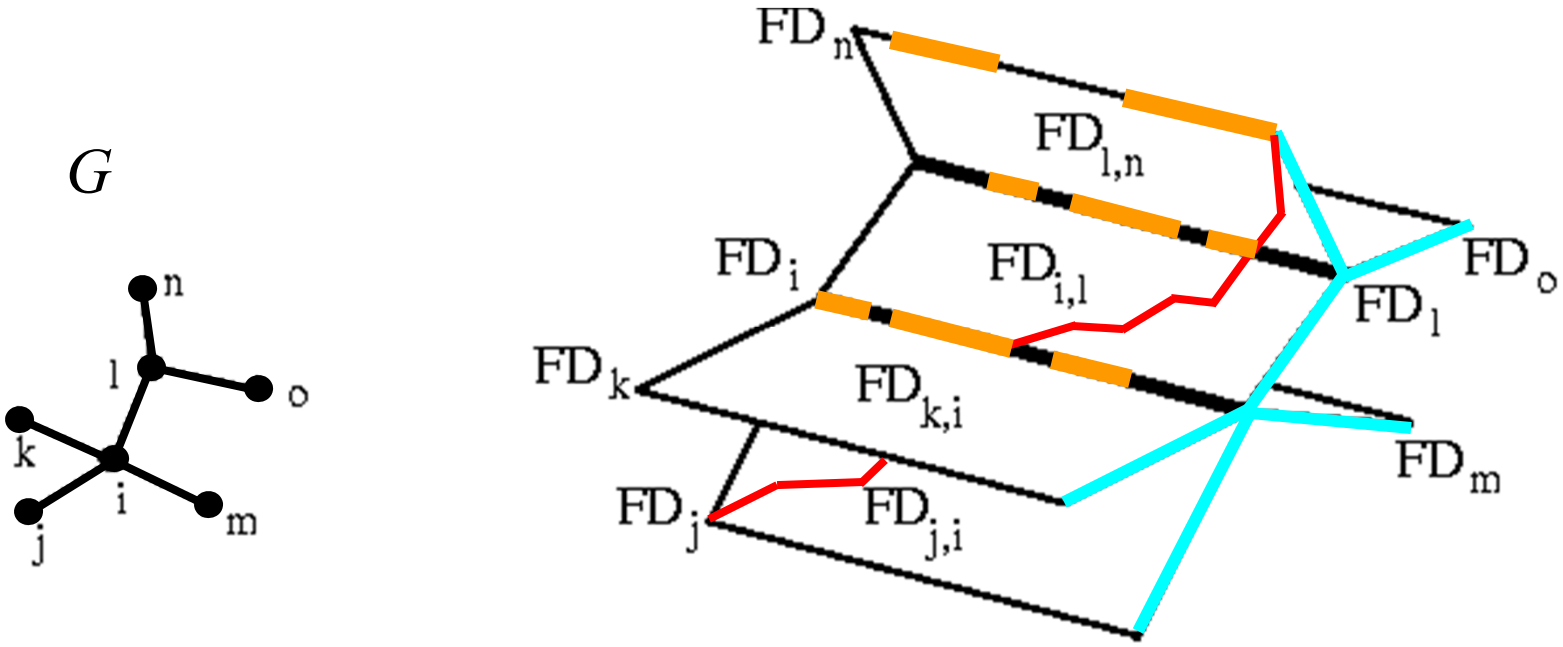
- During the sweep:
  - Update reachable points Dijkstra-style
  - Use a data structure which supports reachability queries in the free space surface

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Backtracking



- After the sweep:
  - Construct a monotone path via backtracking

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.

[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.

[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Map-Matching

- Algorithm for decision problem takes O(mn log(mn)) time and O(mn) space.
- Optimization problem with parametric search: $O(mn \log^2(mn))$ time

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.