


Theorem: Any binary tree with n internal nodes has $\leq n+1$ leaves.

Proof: by induction on n .

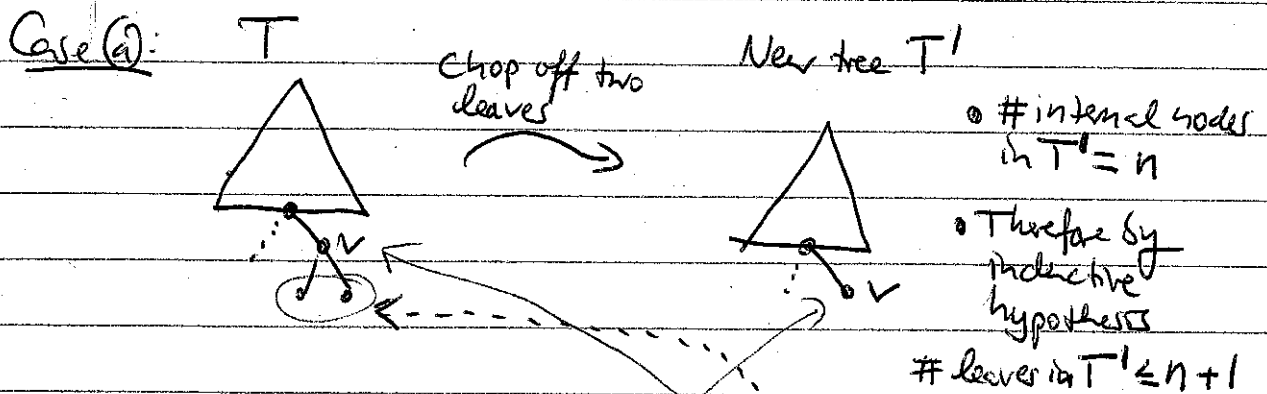
Base Case $n=1$: 

Step: $n \rightsquigarrow n+1$

Let T be a tree with $n+1$ internal nodes.

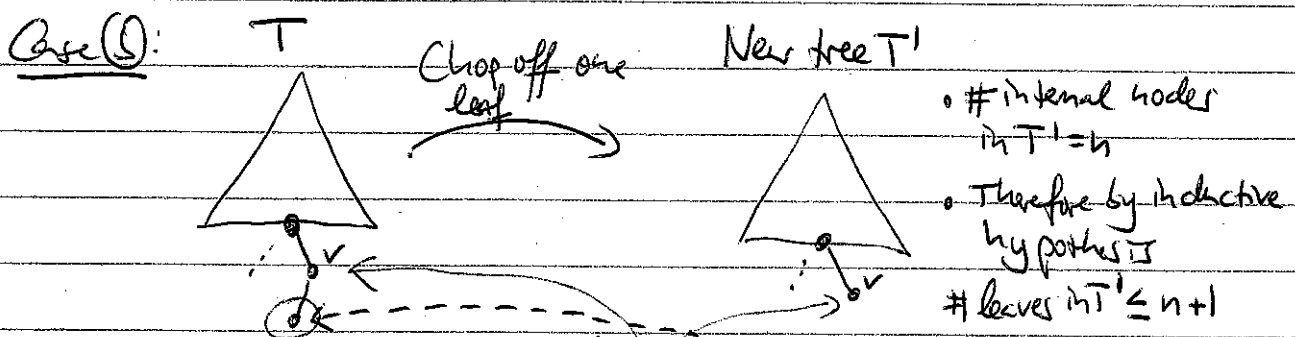
Find a node v in T that has

either (a) two leaves  or (b) one leaf 



$$\Rightarrow \# \text{ leaves in } T \leq \# \text{ leaves of } T' - 1 + 2$$

$$\leq n+1 - 1 + 2 = n+2 \quad \checkmark$$



$$\Rightarrow \# \text{ leaves in } T \leq \# \text{ leaves of } T' - 1 + 1$$

$$\leq n+1 - 1 + 1 = n+1 \leq n+2 \quad \checkmark$$