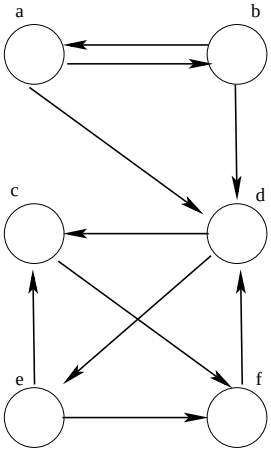
CMPS 2200 Introduction to Algorithms – Fall 15

10/28/15

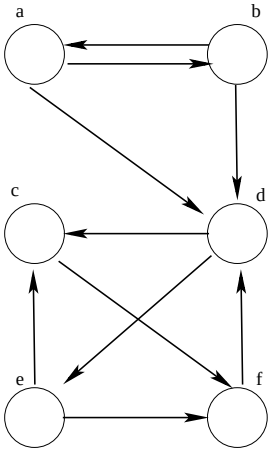
7. Homework

Due **11/5/15** at the beginning of the lab

Graph $G = (V, E)$



Graph $G = (V, E)$



1. **Adjacency matrix and lists (8 points)**
   Give the adjacency matrix representation and the adjacency lists representation for graph G above; note that above are two copies of graph G. Assume that vertices (e.g., in adjacency lists) are ordered alphabetically.

2. **DFS, BFS (8 points)**
   Run BFS and DFS on graph G above, starting on vertex a. Assume that vertices are ordered alphabetically in the adjacency lists.

   Write the visit times (and for DFS the discover and finish times) into the vertices. Draw the tree edges into the graphs and, for both BFS and DFS, show how the tree is stored in the predecessor array. For BFS show the evolving queue. For DFS also show the edge classifications of all edges in the graph.

3. **Cycles (8 points)**
   Let $G = (V, E)$ be a directed graph given in adjacency lists. Let $n = |V|$ and $m = |E|$.

   (a) Describe an efficient algorithm that determines whether G has a cycle. Analyze the runtime of your algorithm in terms of m and n.

   (b) Let $u \in V$ be a fixed vertex. Describe an efficient algorithm that determines whether G has a cycle that contains u. Analyze the runtime of your algorithm in terms of m and n.

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4. **Greedy Skis (5 points + 5 extra credit points)**
Assume there are n people with heights $p_1, \ldots, p_n$ and there are n skis with heights $s_1, \ldots, s_n$. The task is to assign each person a ski in such a way that the average height difference between the person and their assigned ski is minimized. I.e., the task is to minimize

$$\frac{1}{n} \sum_{i=1}^{n} |p_i - s_{f(i)}| ,$$

where f is the assignment function.

Consider the following two greedy algorithms:

(a) Find the person and the ski with the minimum height difference. Assign this ski to this person. Repeat until every person has a ski.

(b) Sort the persons by height and sort the skis by height. Give the shortest person the shortest ski, and repeat.

One of the algorithms above is correct, the other is incorrect. Find the one that is incorrect and disprove it with a counterexample (i.e., an example input where the greedy solution differs from the optimal solution).

For the correct algorithm, you can get up to five extra credit points if you can provide a (partial) proof of its correctness.