CMPS 2200 Introduction to Algorithms – Fall 15

9/22/15

4. Homework

Due **10/1/15** at the beginning of the lab

1. **Recursion tree (8 points)**

   For the recurrences below, use the recursion tree method to find a good guess of what they could solve to asymptotically. Assume $T(1) = 1$.

   (a) (4 points) $T(n) = 4T(n/3) + n^2$

   (b) (4 points) $T(n) = 8T(n/2) + n^3$

2. **3-way mergesort (4 points)**

   Consider the following variant of mergesort, where the first call is `3wayMergesort(0,n-1,A)` to sort the array $A[0..n-1]$.

```
int 3wayMergesort(int i, int j, int[] A){
  // Sort A[i..j]
  if(j-i<=1)
    return;

  int l = (j-i)/3;
  3wayMergesort(i,i+l, A);
  3wayMergesort(i+l+1,i+2*l,A);
  3wayMergesort(i+2*l+1,j,A);
  merge(i,i+l+1,i+2*l+1); // Merges all three sub-arrays in linear time
}
```

   Set up a runtime recurrence for `3-way mergesort` above. Then solve the recurrence using the method of your choice. What is the runtime of `3wayMergesort`?

3. **Divide-and-conquer (5 points)**

   Let A be a sorted array of n distinct integers. Give pseudo-code for a divide-and-conquer algorithm that decides whether there is an index i such that $A[i] = i$. Your algorithm should run in $O(\log n)$ time. Give a runtime recurrence for your algorithm and argue why the runtime is $O(\log n)$. *(Hint: How many subproblems are you allowed to recurse into, in order to get achieve the required runtime?Also, remember that the array contains distinct integers.)*

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4. **Strassen's Algorithm (4 points)**
Apply Strassen's algorithm to compute

$$\begin{pmatrix} 0 & 2 & 1 & 2 \\ 3 & 1 & 0 & 1 \\ 1 & 0 & 5 & 2 \\ 2 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 0 \\ 1 & 3 & 1 & 2 \\ 3 & 4 & 1 & 2 \end{pmatrix}$$

The recursion should exit with the base case $n = 1$, i.e., 2×2 matrices should recursively be computed using Strassen's algorithm. In order to save you some work, you may assume that the following is a partial solution and you only have to fill in the missing values by using Strassen's algorithm:

$$\begin{pmatrix} 11 & 15 & 3 & 6 \\ 8 & 9 & 4 & 5 \\ & & 8 & 15 \\ & & 4 & 6 \end{pmatrix}$$