CMPS 2200 Introduction to Algorithms – Fall 15

9/2/15

1. Homework

Due **9/10/15** at the beginning of the lab

1. **Big-Oh ranking (10 points)**
   Rank the following eleven functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

   $$\sqrt[3]{n}\ ,\ 2^n\ ,\ \log\sqrt{n}\ ,\ n^3\ ,\ 2^{n+2}\ ,\ \log\log n\ ,\ 3n+5\ ,\ 2^{2n}\ ,\ \log n\ ,\ 4^n\ ,\ \sqrt{n}\ ,$$

   Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

   $$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

   if the limits exist; where f and g' are the derivatives of f and g, respectively.

2. O, Ω, Θ **(9 points)**

   Show using the definitions of big-Oh, Ω, and Θ:

   (a) (4 points) $4n^6 + 3n^3 + n \in \Theta(n^6)$

   (b) (2 points) $n + 5 \notin \Omega(n^2)$

   (c) (3 points) If $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$, prove that

   $$f_1(n) + f_2(n) \in O(\max(g_1(n), g_2(n)))\ .$$

3. **Code snippets (5 points)**

   Give the Θ-runtime for the code snippets below, depending on n. Justify your answer.

   (a) (2 points)

   ```
   for(i=n*n; i>=1; i=i/3)
           print(" ");
   ```

   (b) (3 points)

   ```
   for(i=2; i<=n; i=i*i*i)
           print(" ");
   ```