CMPS 2200 Introduction to Algorithms – Fall 14

11/25/14

Extra Credit Homework

Due **12/4/14** at the beginning of class

This homework is NOT mandatory. You are allowed to turn in homeworks in groups of two.
24 points will count as 100%, i.e., you only need 14.4 points to reach 60%.

1. **Median computation (6 points)**
   Suppose arrays A and B are **both sorted** and contain n elements each. The task is to develop a (deterministic) divide-and-conquer algorithm to find the median of $A \cup B$ in $O(\log n)$ time.

   - Assume your divide-and-conquer approach is to split the problem into halves. What exactly does that mean in this case?
   - In order to arrive at an algorithm with $O(\log n)$ runtime, what runtime recurrence would the algorithm have? This should give you an idea on (i) how many recursive calls you can have and (ii) how much time you have for the dividing and combining steps of the algorithm.
   - In the combine step you should be able to discard large parts of the input arrays. Argue why. *(Hint: Make up examples with real numbers in order to better understand what is going on.)*
   - Describe the algorithm in words. Shortly argue why the runtime is $O(\log n)$.

2. **To be or not to be ... in NP (6 points)**
   Which of the problems below are in NP, and which are not? Either justify why the problem is not in NP, or show that it is in NP by sketching an appropriate algorithm and its runtime.

   (a) Given two strings A and B of length m and n, respectively. Compute a longest common subsequence for A and B.

   (b) Given a positive integer a, is a composite, i.e., the product of two integers greater than 1?

   (c) Given an undirected graph $G = (V, E)$, two vertices $s, t \in V$, and a positive integer k. Is there a *simple* path from s to t in G that contains at least k edges?

3. **Fun with reductions (6 points)**
   Suppose Π_1 and Π_2 are decision problems and Π_1 is polynomial time reducible to Π_2, so, $\Pi_1 \leq \Pi_2$. Please answer each of the questions below, and justify your answers.

   (a) If $\Pi_1 \in P$ does this imply that $\Pi_2 \in P$?

   (b) If $\Pi_2 \in NP$ does this imply that $\Pi_1 \in NP$?

   (c) If $\Pi_1 \in NP$ does this imply that Π_2 is NP-complete?

   (d) If Π_1 is NP-complete and $\Pi_2 \in P$, what does this imply?

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4. Weighted Clique **(6 points)**
Consider the following problem:

> Weighted Clique
> **Given:** An undirected graph $G = (V, E)$ with positive integer edge weights $w : E \to \mathbb{R}$, and an integer $S > 0$.
> **Task:** Does G contain a clique C with total weight (= sum of edge weights) at least S?

Show that Weighted Clique is NP-complete by showing:

(a) that Weighted Clique is in NP, and

(b) by giving a polynomial-time reduction from Clique (so, show Clique $\leq$ Weighted Clique)

5. 2-TSP **(6 points)**
Consider the following problem:

> 2-TSP
> **Given:** An undirected graph G with positive integer edge weights, and an integer $S > 0$.
> **Task:** Does G contain two closed tours such that each tour contains at least two vertices, both tours together visit every vertex in V exactly once, and the total sum of all edge weights on both tours is at most S?

Show that 2-TSP is NP-complete by showing:

(a) that 2-TSP is in NP, and

(b) by giving a polynomial-time reduction from TSP (so, show TSP $\leq$ 2-TSP)