CMPS 2200 Introduction to Algorithms – Fall 14

9/16/14

3. Homework

Due **9/25/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **1, 2, 3, 4, 5 (6 points)**
   (a) Describe all (valid) red-black trees that store the numbers $1, 2, 3, 4, 5$.
   (b) Describe all (valid) 2-3-4 trees that store the numbers $1, 2, 3, 4, 5$.
   (c) Now, for every $k \geq 3$, describe all valid B-trees with minimum degree k.

2. **B-tree-search using binary search (4 points)**

   Consider changing B-Tree-Search to use **binary search** instead of linear search on the key.

   (a) What is the number of disk accesses?
   (b) What is the CPU time? Show that the CPU time is only $O(\log n)$, which is independent of k.

3. **Recursion tree (4 points)**
   For the recurrence below, use the recursion tree method to find a good guess of what they could solve to asymptotically (i.e., in big-Oh terms). Assume $T(1) = 1$. You may need to use that $(a^b)^c = a^{b \cdot c} = (a^c)^b$.

$$T(n) = 3T(\frac{n}{2}) + n^4 \text{ for } n \geq 2$$

4. **Induction (4 points)**
   Let $T(1) = 2$ and $T(n) = 3T(n/2) + n^4$ for $n \geq 2$. Use induction to prove that $T(n) \leq 2n^4$ for all $n \geq 1$.

5. **Divide and Conquer (6 points)**
   Let $A[0..n-1]$ be an array of n numbers. A number in A is a *majority element* if A contains this number at least $\lfloor n/2 \rfloor + 1$ times. So for example, the array $A = \{4, 3, 2, 3, 1, 3, 3, 6, 3, 5, 3, 3\}$ has 3 as its majority element.

   Write a divide-and-conquer algorithm that determines whether a given array $A[0..n-1]$ contains a majority element, and if so, returns it. Your algorithm should run in $O(n \log n)$ time. You are **not** allowed to sort the array.

   Set up and solve a recurrence relation for the runtime of your algorithm.

   Hint: Start by applying the generic divide-and-conquer approach. Try to divide by two. Then try to combine the results. From the given runtime you should be able to guess how much time you are allowed to spend for dividing and combining.