CMPS 2200 Introduction to Algorithms – Fall 14

8/29/14

1. Homework

Due **9/11/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Big-Oh ranking (10 points)**
   Rank the following ten functions by order of growth, i.e., find an arrangement $f_1, f_2, \ldots$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

   $$3^n \ , \ \log n \ , \ n^2 \ , \ n^2 \log n \ , \ 2^n \ , \ \sqrt{n} \ , \ n^3 \ , \ 4^{\log_2 n} \ , \ \log\log n \ , \ n \ , \ 3^{n+2} \ ,$$

   Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

   $$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

   if the limits exist; where f and g' are the derivatives of f and g, respectively.

2. O, Ω, Θ **(10 points)**

   Show using the definitions of big-Oh, Ω, and Θ:

   (a) (4 points) $8n^3 + 4n - 5 \in \Theta(n^3)$

   (b) (2 points) $2n^2 + 7 \notin \Omega(n^3)$

   (c) (4 points) Which of the following statements is true? Justify your answers with either a proof or a counter example.
   If $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$ then

   i. $f_1(n) + f_2(n) \in O(\min(g_1(n), g_2(n)))$.

   ii. $f_1(n) + f_2(n) \in O(\max(g_1(n), g_2(n)))$.

3. **Code snippet (4 points)**

   Give the Θ-runtime for the code snippet below, depending on n. Justify your answer.

```
for(i=1; i<=n*n; i=i+4)
  for(j=n; j>=1; j=j/4)
        print(" ");
```