CMPS/MATH 2170 -- Fall 2015
Discrete Mathematics

Probability distributions

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Rolling two dice

Sum of two fair six-sided dice:

S = {1,2,3,4,5,6} x {1,2,3,4,5,6}

$X((s_1,s_2)) = s_1+s_2$

x	(X=x)
2	{(1,1)}
3	{(1,2),(2,1)}
4	{(1,3),(2,2),(3,1)}
5	{(1,4),(2,3),(3,2),(4,1)}
6	{(1,5),(2,4),(3,3),(4,2),(5,1)}
7	{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}
8	{(2,6),(3,5),(4,4),(5,3),(6,2)}
9	{(3,6),(4,5),(5,4),(6,3)}
10	{(4,6),(5,5),(6,4)}
11	{(5,6),(6,5)}
12	{(6,6)}

Expected value:

$$E(X) = \sum_{x=2}^{12} xP(X = x)$$

Use linearity of expectation:

$$X_1, X_2 : S \to \mathbb{R}$$
$$X_1\big((i,j)\big) = i$$
$$X_2\big((i,j)\big) = j$$
$$\Rightarrow X = X_1 + X_2$$

$$E(X) = E(X_1 + X_2)$$
$$= E(X_1) + E(X_2)$$
$$= \frac{7}{2} + \frac{7}{2} = 7$$

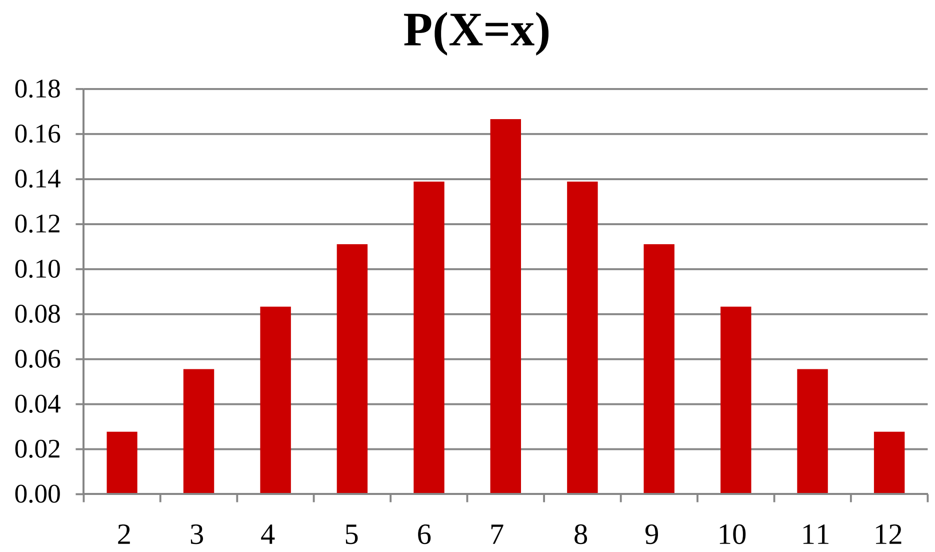
Rolling two dice

Sum of two fair six-sided dice:

S = {1,2,3,4,5,6} x {1,2,3,4,5,6}

$X((s_1,s_2)) = s_1+s_2$

x	P(X=x)	(X=x)
2	1/36	{(1,1)}
3	2/36	{(1,2),(2,1)}
4	3/36	{(1,3),(2,2),(3,1)}
5	4/36	{(1,4),(2,3),(3,2),(4,1)}
6	5/36	{(1,5),(2,4),(3,3),(4,2),(5,1)}
7	6/36	{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}
8	5/36	{(2,6),(3,5),(4,4),(5,3),(6,2)}
9	4/36	{(3,6),(4,5),(5,4),(6,3)}
10	3/36	{(4,6),(5,5),(6,4)}
11	2/36	{(5,6),(6,5)}
12	1/36	{(6,6)}



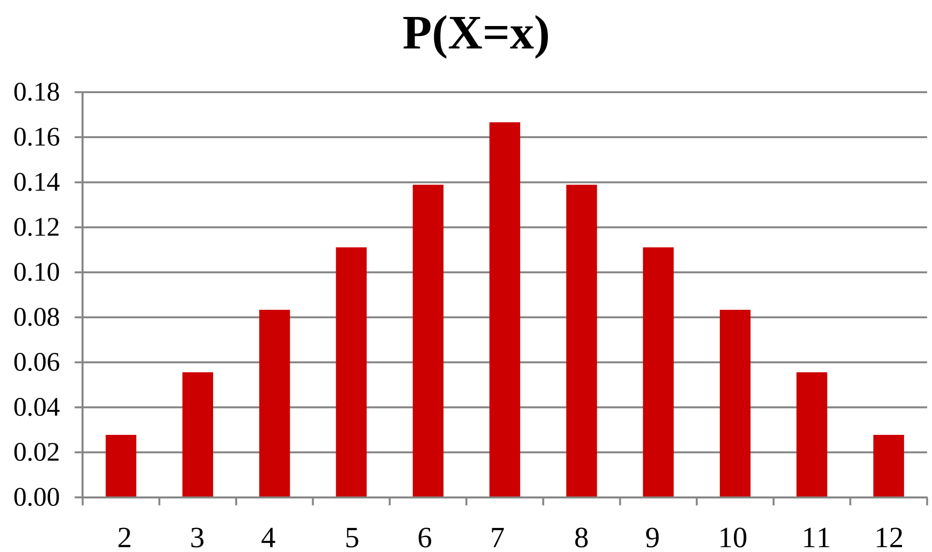
Probability density function or probability distribution: p(x) = P(X=x)

Rolling two dice

Expected value: $E(X) = \sum_{x=2}^{12} x\, p(x) = 7$

Variance: $V(X) = E(X^2) - E(X)^2 =$

x	p(x)	x p(x)	x^2 p(x)
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36
Sum	36/36 = 1	252/36 = 7	1974/36 = 54.833



Probability density function or probability distribution: p(x) = P(X=x)

Rolling two dice

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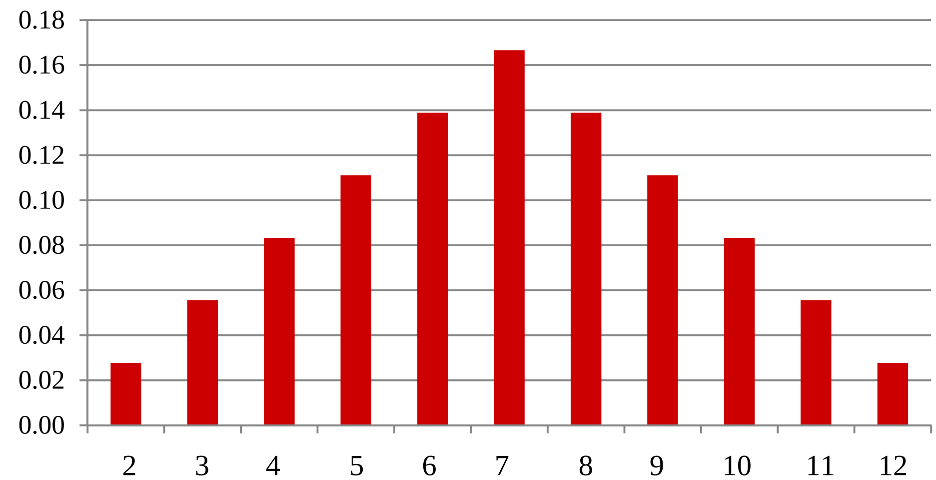
Expected value: $E(X) = \sum_{x=2}^{12} x\, p(x) = 7$

Variance: $V(X) = E(X^2) - E(X)^2 =$

$$= \frac{1974}{36} - 7^2 = \frac{987}{18} - 49 = 5.833$$

$\sigma = \sqrt{5.833} = 2.42$

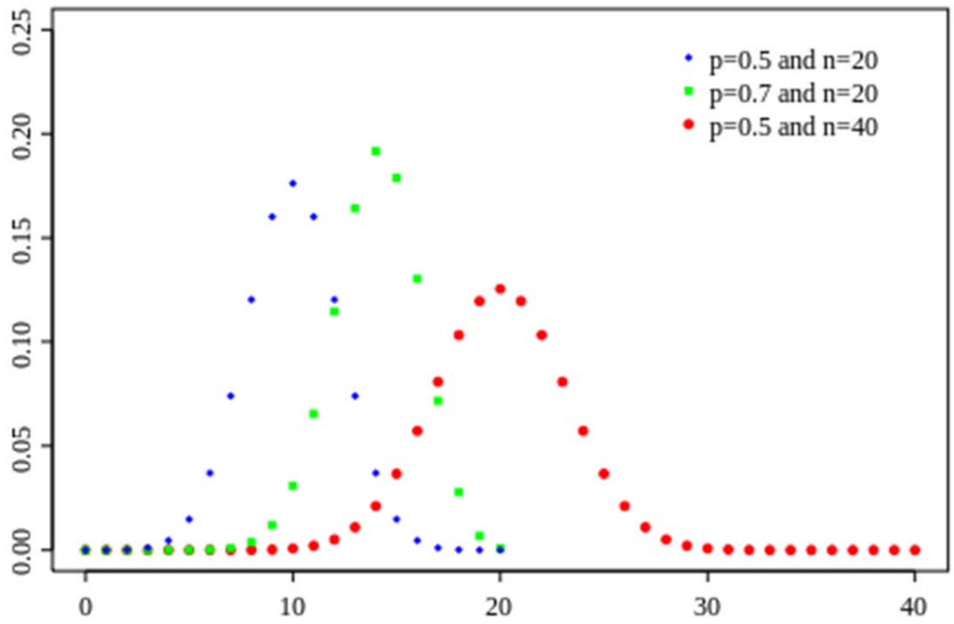


Probability density function or probability distribution: p(x) = P(X=x)

Binomial Distribution

$X =$ Number of successes in a sequence of n independent Bernoulli trials, with individual success probability p

$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$



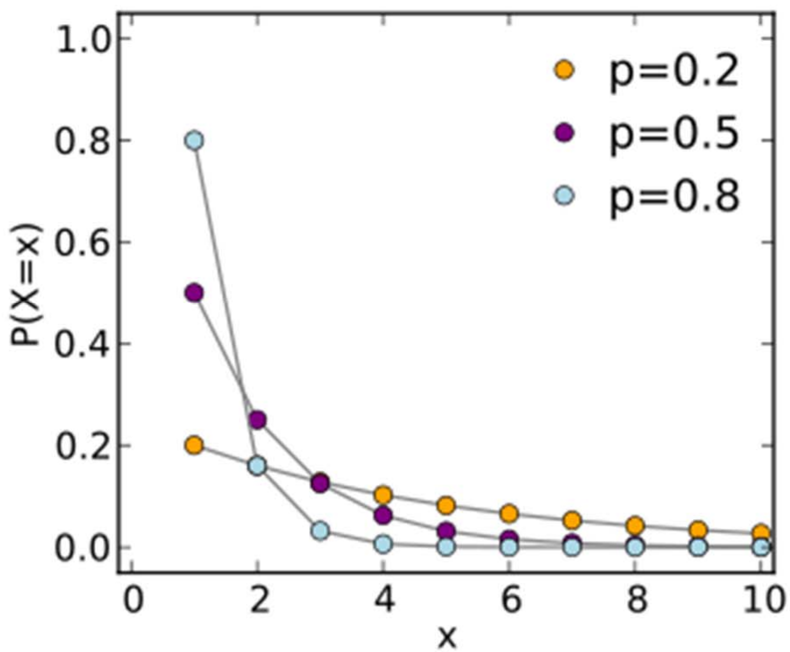
Expected value: $E(X) = np$

Variance: $V(X) = np(1-p)$

Geometric Distribution

X = Number of independent Bernoulli trials needed to get the first success; where the individual success probability is p

$P(X=k) = (1-p)^{k-1}p$



Expected value: $E(X) = 1/p$

Variance: $V(X) = (1-p)/p^2$