CMPS/MATH 2170 Discrete Mathematics – Fall 15

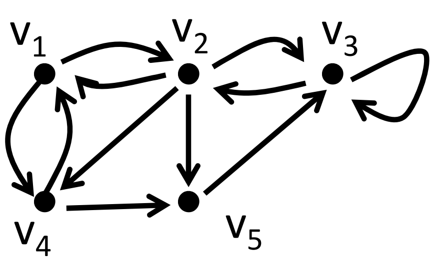
11/24/15

Extra Credit Homework

Due **12/3/15** at the beginning of the lab

1. **Relations (8 points)**
   - (a) (4 points) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ iff a has the same first name as b.
   - (b) (4 points) Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings of length three or more that agree except perhaps in their first three bits is an equivalence relation on the set of all bit strings of length three or more.

2. **Graph representations (6 points)**
   Consider the graph $G = (V, E)$ below:



   - (a) Specify the set of vertices V.
   - (b) Specify the set of edges E.
   - (c) Give the in-degree and the out-degree for each vertex.
   - (d) Verify that the handshaking lemma holds.
   - (e) Give the adjacency matrix representation for this graph.
   - (f) Give the adjacency lists representation for this graph.

3. **Graphs (6 points)**
   - (a) (3 points) Let $G = (V, E)$ be a (simple and undirected) graph. Let D be the maximum degree of all vertices, and let d be the minimum degree of all vertices. Show that $d \leq 2|E|/|V| \leq D$.
   - (b) (3 points) Describe the adjacency matrix of a graph with k connected components when the vertices of the graph are listed so that vertices in each connected component are listed successively.

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4. **Pitchers (4 points)**
   Suppose you have two pitchers, one can hold three cups the other five cups. You may apply the following operations on either pitcher: Fill it with lemonade, empty it, or transfer the lemonade to the other pitcher.

   Use a path in a directed graph to show that you can end up with a pitcher containing exactly one cup of lemonade. *(Hint: Use (a, b) to indicate that a cups are in the first jug and b cups are in the second jug. Let (a, b) be the vertices, and represent each operation as an edge.)*

5. **Trees (4 points)**
   Let G be a simple graph. Show that G is a tree if and only if (i) G is connected and (ii) the deletion of any of its edges produces a graph that is not connected.