CMPS/MATH 2170 Discrete Mathematics – Fall 15

11/11/15

9. Homework

Due **11/19/15** at the beginning of the lab

Please provide short explanations for your answers, such as noting whether you use combinations or permutations, with or without repetition.

1. **Counting (10 points)**
   (a) A committee of three is chosen from a group of 20 people. How many different committees are possible if
      i. (2 points) the committee consists of a president, vice president, and treasurer?
      ii. (2 points) there is no distinction among the three members of the committee?

   (b) (2 points) There are nine empty seats in a theater, and five customers need to find places to sit. How many different ways can these five seat themselves?

   (c) How many different ways are there to choose ten pastries from the 20 varieties at a bakery...
      i. (2 points) ... such that no two pastries are of the same variety?
      ii. (2 points) ... if there are no restrictions? (I.e., multiple pastries could be of the same variety.)

2. **String Counting (8 points)**
   (a) (2 points) How many permutations of the letters TULANE contain the string TAN?

   (b) (2 points) How many three-letter strings can be made from the word VICTORY?

   (c) (2 points) How many strings can be made from the word BANANARAMA?

   (d) (2 points) Let n be a positive integer. How many (non-empty) bit strings are there of length at most n?

3. **Pigeonholes (6 points)**
   (a) (2 points) Show that in a set of $n + 1$ different integers, at least two are congruent modulo n.

   (b) (2 points) Show that every 68-element subset of $\{1, 2, 3, \ldots, 100\}$ contains three consecutive integers.

   (c) (2 points) How many students have to minimally be in a class to ensure that there are at least three students whose last name starts with the same letter?

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4. **Binomial Theorem (6 points)**

(a) (1 point) What is $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots + \binom{n}{n}$?

(b) (1 point) What is $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots \pm \binom{n}{n}$?

(c) (4 points) Use a counting argument to prove that

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$$

(Hint: Remember, $\binom{n}{i} = \binom{n}{n-i}$*)*