10/28/15

7. Homework
Due **11/5/15** at the beginning of the lab

1. **Climbing a ladder (8 points)**
   Consider climbing a ladder with n rungs. The rungs are spaced such that you can climb one rung, two rungs, or three rungs at a time. Let $r(n)$ be the number of different ways to climb a ladder with n rungs. For example, $r(2) = 2$ because one can climb a 2-rung ladder either as $1 + 1$ rungs or as 2 rungs, which are two different climbing patterns.

   (a) (3 points) Give the values of $r(1), r(3), r(4)$; justify your answers.

   (b) (4 points) Develop a recursive formula for $r(n)$. Explain your answer. *(Hint: This will look similar to the Fibonacci numbers, with multiple base cases and more than one recursive "call".)*

   (c) (1 point) What is $r(7)$?

2. **Recursive definitions (6 points)**
   Give a recursive definition of:

   (a) (2 points) the **sequence** $\{a_n\}_{n \in \mathbb{N}}$ if $a_n = 3n + 5$.

   (b) (2 points) the **set** of positive integer powers of 3.

   (c) (2 points) the **function** $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ where $f(n)$ is the sum of positive integers less or equal to n.

3. **Solve recurrences (6 points)**
   Solve the linear recurrences below.
   (Hint: Compute the roots of the characteristic equation, and then use the initial conditions to solve for α_1 and α_2.)

   (a) (3 points) $a_n = 8a_{n-1} - 16a_{n-2}$ for $n \geq 2$; $a_0 = 2, a_1 = 12$

   (b) (3 points) $a_n = -3a_{n-1} + 4a_{n-2}$ for $n \geq 2$; $a_0 = 5, a_1 = 10$

4. **Quotient and remainder (2 points)**
   Find the quotient ("div") and the remainder ("mod") when:

   (a) (1 point) 23 is divided by 6

   (b) (1 point) -42 is divided by 5

5. **Congruence (3 points)**
   Let $a, b, c > 0, m \geq 2$ be integers. Prove that

   $$a \equiv b \pmod{m} \quad \text{implies that} \quad ac \equiv bc \pmod{mc}.$$

6. **Euclidean algorithm (3 points)**
   Use the Euclidean algorithm to find $gcd(12345, 678)$.