CMPS/MATH 2170 Discrete Mathematics – Fall 15

9/30/15

5. Homework

Due **10/8/15** at the beginning of the lab

1. **Composition I (7 points)**
   Let $f, g : \mathbb{R} \to \mathbb{R}$ with $f(x) = 2x^3 + 3$ and $g(x) = 4x - 1$.
   (a) (1 point) Find $f \circ g$ and $g \circ f$.
   (b) (4 points) Show that f and g are bijective.
   (c) (2 points) Find f^{-1}, g^{-1}, $(f \circ g)^{-1}$ and $(g \circ f)^{-1}$.
   (d) (1 extra credit point) Generally, $f \circ g \neq g \circ f$. Can you give functions f and g, with $f \neq g$ and both different from the identity function, such that $f \circ g = g \circ f$?

2. **Composition II (4 points)**
   Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions. Show that if f and g are bijective then $f \circ g$ is bijective.

3. **Cardinality I (6 points)**
   Determine whether the sets below are finite, countably infinite, or uncountable. Justify your answers.
   (a) (2 points) The integers between 0 and 9.
   (b) (2 points) The rational numbers between 0 and 9.
   (c) (2 points) The real numbers between 0 and 9.

4. **Cardinality II (3 points)**
   Give an example of two uncountable sets A and B such that
   (a) (1 point) $A \setminus B$ is finite.
   (b) (1 point) $A \setminus B$ is countably infinite.
   (c) (1 point) $A \setminus B$ is uncountable.

5. **Cardinality III (8 points)**
   Let A, B be two sets such that $A \subseteq B$. Prove or disprove each of the statements below.
   (a) (2 points) If A is uncountable then B is uncountable.
   (b) (2 points) If A is countable then B is countable.
   (c) (2 points) If B is uncountable then A is uncountable.
   (d) (2 points) If B is countable then A is countable.