9/22/15

4. Homework
Due **10/1/15** at the beginning of the lab

1. $\{2\}$ **(4 points)**
   Determine which of these statements are true or false.

   (a) $\{2\} \subseteq \{2\}$

   (b) $\{2\} \subseteq \{\{2\}\}$

   (c) $\{2\} \in \{\{2\}\}$

   (d) $\{2\} \in \{\{\{2\}\}\}$

2. **Set Identity (5 points)**
   Prove that $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

3. **Power Sets (10 points)**

   (a) (2 points) Find $\mathcal{P}(A \times B)$ where $A = \{1, 2\}$ and $B = \{3, 4\}$.

   (b) (4 points) Determine if each of these sets is the power set of a set.

       i. $\emptyset$ ii. $\{\emptyset, 2\}$ iii. $\{\emptyset, \{2\}, \{\emptyset, 2\}\}$ iv. $\{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$

   (c) (4 points) Prove or disprove: If $\mathcal{P}(A) = \mathcal{P}(B)$ then $A = B$.

4. **Onto and One-to-One (3 points)**
   Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is

   (a) one-to-one but not onto

   (b) onto but not one-to-one

   (c) neither one-to-one nor onto.

5. **Strictly Increasing (5 points)**
   A function $f : \mathbb{R} \to \mathbb{R}$ is *strictly increasing* iff $\forall x_1, x_2 \in \mathbb{R} : x_1 < x_2 \to f(x_1) < f(x_2)$.
   Show that every strictly increasing function is one-to-one.

6. **Functions (8 points)**
   Let $f(x) = 3x^3$. For each of the domains and co-domains below, determine whether f is a valid function, and if so whether it is injective, surjective, or bijective.

   (a) $f : \mathbb{R} \to \mathbb{R}$ (b) $f : \mathbb{N} \to \mathbb{N}$ (c) $f : \mathbb{N} \to \mathbb{R}$ (d) $f : \mathbb{R} \to \mathbb{N}$