

3. Homework

Due **9/24/15** at the beginning of the lab

1. Irrational (6 points)

- (a) (3 points) Prove that $\sqrt[3]{2}$ is irrational.
- (b) (3 points) Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

2. Equivalence (7 points)

- (a) (4 points) Prove that $m = n$ or $m = -n$ if and only if $m^2 = n^2$.
- (b) (3 points) Show that the propositions p_1, p_2, p_3, p_4 can be shown to be equivalent by showing that $p_1 \leftrightarrow p_4$, $p_2 \leftrightarrow p_3$, and $p_1 \leftrightarrow p_3$.

3. Cases (3 points)

Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$, for all $a, b, c \in \mathbb{R}$.

4. Existence (2 points)

Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or non-constructive?

5. Even integers (3 points)

Prove that there is no largest even positive integer.