9/9/15

2. Homework

Due **9/16/15** at the beginning of class

1. **Translation I (2 points)**
   Translate this statement into English, where the domain for each variable consists of the real numbers.
   $\forall x \forall y : ((x \geq 0 \land y < 0) \rightarrow x - y > 0)$

2. **Translation II (6 points)**
   Let $M(x, z)$ be the statement that student x has been in movie z, and let $K(x, y)$ be the statement that student x knows student y. Express each of these sentences in terms of M, K, quantifiers and logical connectives, where the domain for x and y consists of all students at Tulane and for z consists of all movies.

   (a) Everybody knows somebody.

   (b) Every student at Tulane knows another student at Tulane who has been in some movie.

   (c) At least two students at Tulane have been in "22 Jump Street".

3. **Negation (4 points)**

   (a) (2 points) Give a simple English sentence that negates "Everybody knows somebody.". Do this by taking the logical formula of the sentence from the question above, negate it, simplify it such that there is no negation symbol in front of a quantifier, and finally translate it to English.

   (b) (2 points) Rewrite the statement below such that all negation symbols immediately precede predicates P, Q, R, S:
   $\neg[\exists x \forall y : (P(x) \rightarrow (Q(x, y) \land \forall z : R(x, y, z)))]$

4. **Squares (3 points)**
   Express the following statement using predicates, quantifiers, logical connectives, and mathematical operators, where the domain consists of all integers:

   There is a positive integer that is not the sum of two squares.

5. **Conclusions I (3 points)**
   Consider the following set of premises:

   (a) All insects have six legs. (b) Dragonflies are insects.
   (c) Spiders do not have six legs. (d) Spiders eat dragonflies.

   What relevant conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

6. **Conclusions II (3 points)**

   Consider the following set of premises:

   (a) If it is not foggy and it does not rain, then the sailing race will be held and the lifesaving demonstration will go on.

   (b) If the sailing race is held, then the trophy will be awarded.

   (c) The trophy was not awarded and it is not foggy.

   Use rules of inference to show that if these premises imply the conclusion "It rained."

7. **Direct Proof (3 points)**
   Use a direct proof to show that the sum of three odd integers is odd.