CMPS/MATH 2170 Discrete Mathematics – Fall 15

10. Homework

Due **12/3/15** at the beginning of the lab

1. **Dice game (6 points)**

   (a) (2 points) Consider playing the following game: You roll one loaded eight-sided die, where the probability of rolling an eight is 3/10 and the probability of rolling any other number is 1/10. When rolling an eight you win \$25 when rolling any other number you lose \$10. Compute the expected win/loss of this game.

   (b) (2 points) Now consider playing the same game but rolling two loaded dice of the same type. For each eight included in your result you win \$25 and for every other number you lose \$10. Use linearity of expectation to compute the expected win/loss of this game.

   (c) (2 points) Now consider playing the same game but rolling k loaded dice. Use linearity of expectation to compute the expected win/loss of this game.

   Clearly describe the sample space and the random variables you use. Half of the points will be given for correct notation. (The point of this exercise is to learn the notation, not just to get the intuition right.)

2. **Poker (8 points)**

   Consider playing a 4-card poker game, for which one pays \$1 to play, draws 4 cards at random, and wins as follows if the hand is of a particular type:

Royal flush	\$200	(straight flush with ace as high card)
Four of a kind	\$50	(e.g., four threes of all suits)
Straight flush	\$25	(four cards consecutively in order of the same suit, e.g., $4, 5, 6, 7$ of hearts)
Three of a kind	\$6	(e.g., three fives of different suits)
Straight	\$4	(four cards consecutively in order of any suit, e.g., $3, 4, 5, 6$ with 3 of hearts, 4 of spades, 5 of clubs, 6 of hearts)
Flush	\$3	(all four cards of the same suit, e.g., 4, 6, 10, jack of hearts)
Pair of jacks or better	\$1	(e.g., two jacks, two queens, two kings, or two aces of any suit)

   Note that for a hand the highest type counts, e.g., three jacks and two kings are a full house and not two pairs. Also note that for a straight, or a straight flush, the ace may count as the high card (after the king), or as the low card (before the two). But wraparound, such as king, ace, 2, 3 is not allowed.

   (a) (6 points) Compute the probability for each of the hand types listed above.

   (b) (2 points) Compute the expected win/loss for this game. Note that playing the game costs \$1, and hence \$1 has to be subtracted from each winning hand (e.g., the win for a straight flush is really \$24).

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3. **Heads and Tails (3 points)**
   Consider tossing a biased coin, where the probability that heads comes up is p. If the coin is tossed n times, what is the probability that:

   (a) Only tails come up?

   (b) At least one head comes up?

   (c) At most one head comes up?

4. **Variance (3 points)**
   Let X be a random variable. Show that $V(X) = E(X^2) - E(X)^2$.

5. **Bayes (4 points)**
   A test for steroids is given to football players, and 98% of the players taking steroids test positive while 12% of the players not taking steroids test positive. Suppose that 5% of players take steroids. What is the probability that a player who tests positive takes steroids?