9/2/15

1. Homework
Due **9/9/15** at the beginning of class

1. **Propositions (3 points)**
   Consider the following propositions:
   s: It is sunny; r: It rains; c: There are clouds in the sky
   Write the propositions below using s, r, c and logical operators.

   (a) If it rains then there are clouds in the sky and it is not sunny.

   (b) It is sunny even though there are clouds in the sky.

   (c) It rains only if there are clouds in the sky.

2. **Equivalences (6 points)**
   Consider the following equivalence: $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$

   (a) Show the equivalence using truth tables.

   (b) Show the equivalence by establishing a sequence of equivalences. You may use all equivalences in Table 6 and the first equivalence in Table 7. Show your work by annotating every step.

3. **NOR (7 points)**
   We showed in class that $\{\land, \lor, \neg\}$ is functionally complete, i.e., any Boolean function can be expressed using a combination of $\land, \lor, \neg$.

   (a) (2 points) Show that $\{\lor, \neg\}$ is functionally complete.

   (b) Consider the NOR operator $\downarrow$ which is defined using the truth table below.

x	y	$x \downarrow y$
0	0	1
0	1	0
1	0	0
1	1	0

      i. (1 point) Show $x \downarrow y \equiv \neg(x \lor y)$.

      ii. (4 points) Find a compound proposition equivalent to $\neg p$ that uses only $\downarrow$. And find a compound proposition equivalent to $p \lor q$ that uses only $\downarrow$. Now argue that $\{\downarrow\}$ is functionally complete.

4. **Quantifiers (8 points)**

   Justify your answers shortly.

   (a) (4 points) Let $P(x)$ be the statement $x + 1 > 2x$, and let the domain be all integers. Determine the truth values of the following expressions.

        i. $P(0)$ ii. $\forall x : P(x)$ iii. $\exists x : P(x)$ iv. $\forall x : \neg P(x)$

   (b) (1 point) Find a counterexample to: $\forall x : x^2 \geq x$, where the domain is the real numbers.

   (c) (2 points) Give an existential quantification that is true for one domain and false for another domain.

5. **Negating Quantifiers (6 points)**

   (a) (3 points) Consider the sentence: "All German movies are fun."

        i. Express the sentence using quantifiers.
        ii. Negate the quantification, and simplify such that there is no $\neg$ symbol in front of the quantifier.
        iii. Now express this negated sentence in English.

   (b) (3 points) Perform the same three steps (express using quantifiers, negate, express negated sentence in English) for the sentence: "There exists a pig that can swim and catch fish."