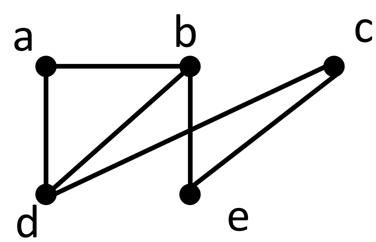
CMPS/MATH 2170 Discrete Mathematics – Fall 14

12/3/14

Some graph problems to study

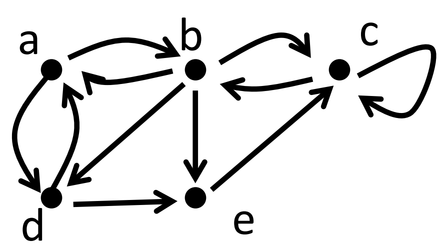
1. **Graph representations**

   (a) Consider the graph below:



   i. Specify the set of vertices V.
   ii. Specify the set of edges E.
   iii. Give the degree for each vertex.
   iv. Verify that the handshaking lemma holds.
   v. Draw the directed graph that can be used to represent this undirected graph.
   vi. Give the adjacency matrix representation for this graph. (Assume vertices are sorted lexicographically.)
   vii. Give the adjacency lists representation for this graph.

   (b) Consider the graph below:



   i. Specify the set of vertices V.
   ii. Specify the set of edges E.
   iii. Give the in-degree and the out-degree for each vertex.
   iv. Verify that the handshaking lemma holds.
   v. Give the adjacency matrix representation for this graph. (Assume vertices are sorted lexicographically.)
   vi. Give the adjacency lists representation for this graph.

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2. **Graphs**

   (a) Let $G = (V, E)$ be a (simple and undirected) graph. Let B be the maximum degree of all vertices, and let A be the minimum degree of all vertices. Show that $A \leq 2|E|/|V| \leq B$.

   (b) Describe the adjacency matrix of a graph with k connected components when the vertices of the graph are listed so that vertices in each connected component are listed successively.

3. **Trees** (harder)

   (a) Let G be a simple graph. Show that G is a tree if and only if (i) G is connected and (ii) the deletion of any of its edges produces a graph that is not connected. *(Hint: Show G is a tree implies (i) and (ii). Then show (i) and (ii) imply that G is a tree.)*

   (b) Show that a connected graph with n vertices has to have at least $n - 1$ edges.

   (c) Use (strong) induction on l to show that for all $l \geq 1$, a full binary tree with l leaves has $2l - 1$ vertices total.

4. **Planar graphs**

   (a) Is the graph in question 1(a) planar? Justify your answer.

   (b) Is there a planar graph with 5 vertices and 12 edges? Justify your answer. What about 5 vertices and 8 edges?

   (c) Does a graph have to be drawn without edge crossings in order to be planar?