

CMPS/MATH 2170 -- Fall 2014
Discrete Mathematics

Probability distributions

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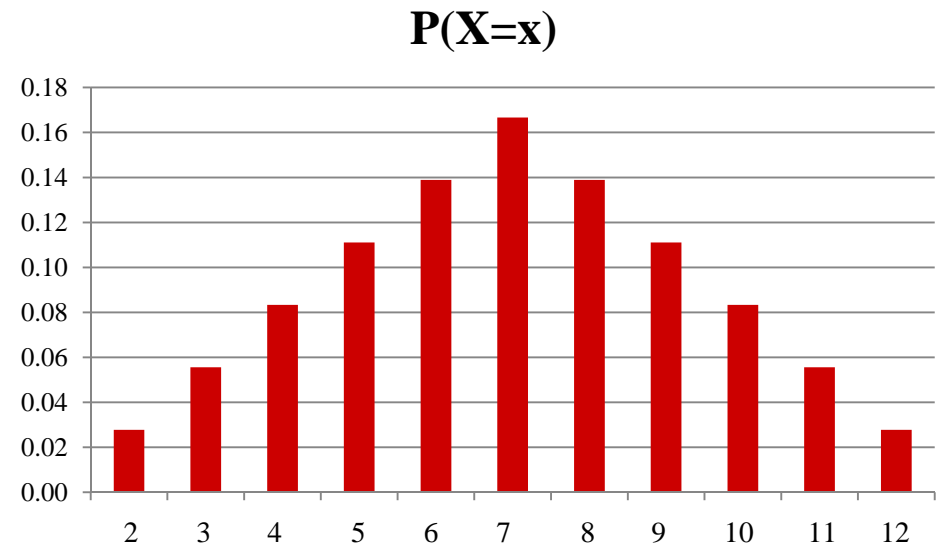
Rolling two dice

Sum of two fair six-sided dice:

$$S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$$

$$X((s_1,s_2)) = s_1 + s_2$$

x	P(X=x)	{X=x}
2	1/36	{(1,1)}
3	2/36	{(1,2),(2,1)}
4	3/36	{(1,3),(2,2),(3,1)}
5	4/36	{(1,4),(2,3),(3,2),(4,1)}
6	5/36	{(1,5),(2,4),(3,3),(4,2),(5,1)}
7	6/36	{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}
8	5/36	{(2,6),(3,5),(4,4),(5,3),(6,2)}
9	4/36	{(3,6),(4,5),(5,4),(6,3)}
10	3/36	{(4,6),(5,5),(6,4)}
11	2/36	{(5,6),(6,5)}
12	1/36	{(6,6)}



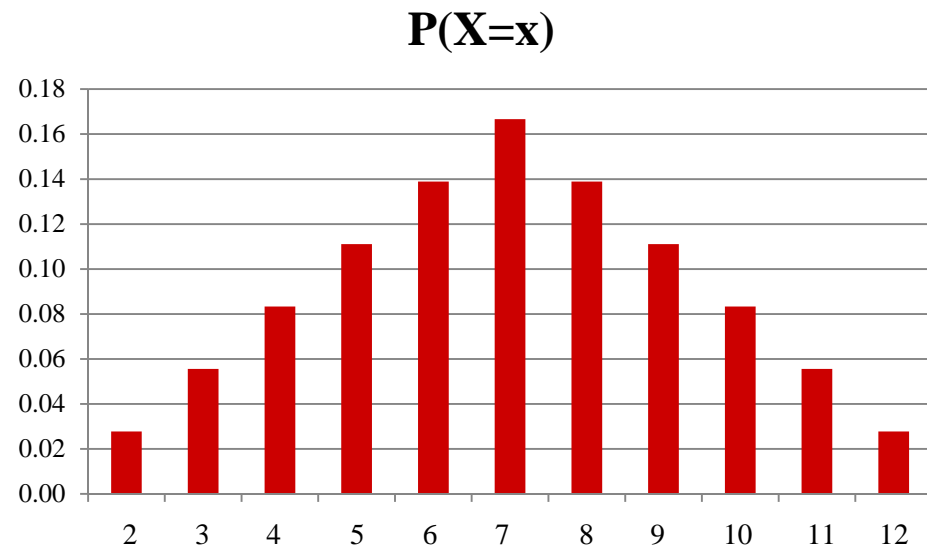
Probability function or probability distribution: $p(x) = P(X=x)$

Rolling two dice

Expected value: $E(X) = \sum_{x=2}^{12} x p(x) = 7$

Variance: $V(X) = E(X^2) - E(X)^2 =$

x	p(x)	x p(x)	x ² p(x)
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36
Sum	36/36 = 1	252/36 = 7	1974/36 = 54.833



Probability function or probability distribution: $p(x) = P(X=x)$

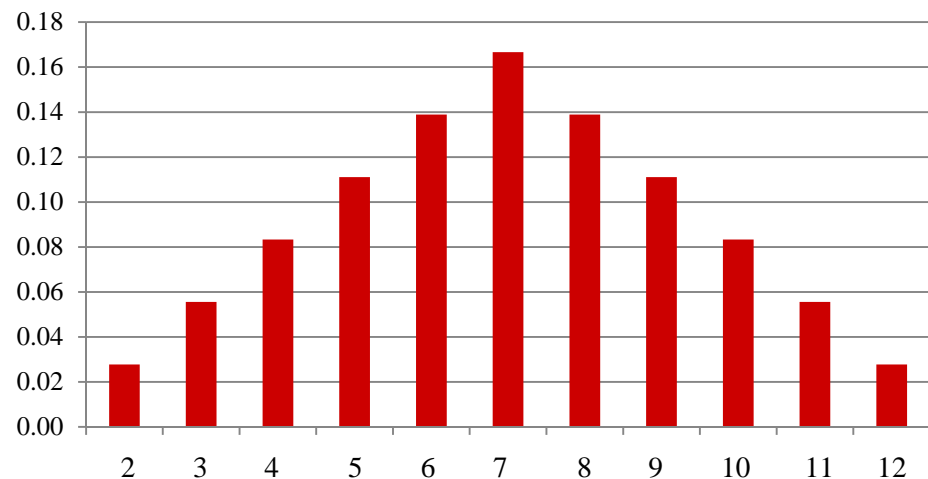
Rolling two dice

Expected value: $E(X) = \sum_{x=2}^{12} x p(x) = 7$

Variance: $V(X) = E(X^2) - E(X)^2 =$
 $= \frac{1974}{36} - 7^2 = \frac{987}{18} - 49 = 5.833$

$\sigma = \sqrt{5.833} = 2.42$

x	p(x)	x p(x)	x ² p(x)
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
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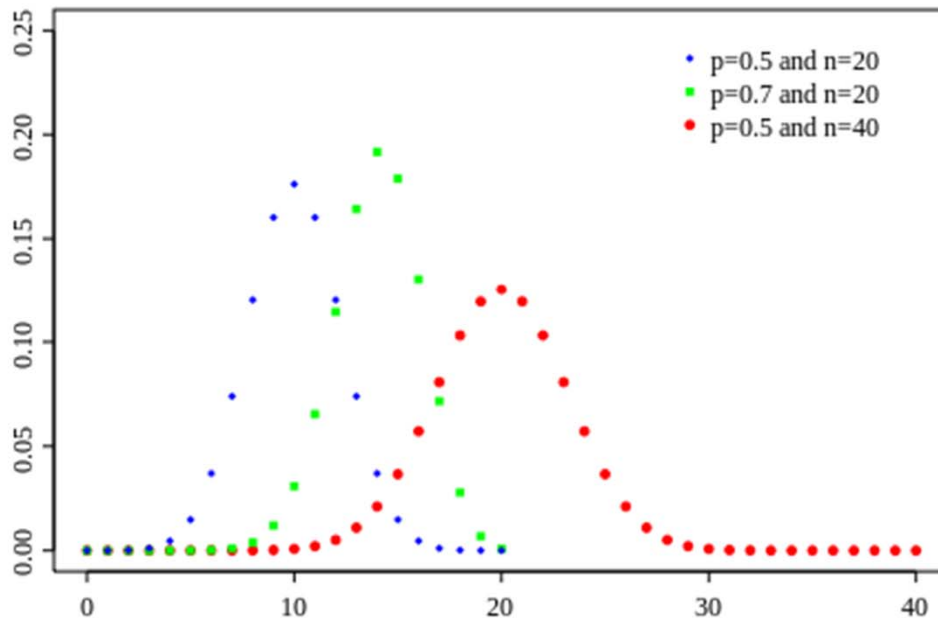


Probability function or probability distribution: $p(x) = P(X=x)$

Binomial Distribution

X = Number of successes in a sequence of n independent Bernoulli trials, with individual success probability p

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$



Expected value: $E(X) = np$

Variance: $V(X) = np(1-p)$