CMPS/MATH 2170 Discrete Mathematics – Fall 14

11/19/14

9. Homework

Due Friday **12/5/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two.

1. **Expected values of dice (8 points)**
   (a) (3 points) Consider playing the following game: You roll one loaded six-sided die, where the probability of rolling a one is 1/15, the probability of rolling a two is 2/15 and the probability of rolling any other number is 3/15. When rolling a one you win \$10, when rolling a two you win \$5, and when rolling any other number you lose \$3. Compute the expected win/loss of this game.
   (b) (3 points) Now consider playing the same game but rolling two loaded dice of the same type. For each one included in your result you win \$10, for each two you win \$5 and for every other number you lose \$3. Use linearity of expectation to compute the expected win/loss of this game.
   (c) (2 points) Now consider playing the same game but rolling k loaded dice. Use linearity of expectation to compute the expected win/loss of this game.

   Clearly describe the sample space and the random variables you use. Half of the points will be given for correct notation. (The point of this exercise is to learn the notation, not just to get the intuition right.)

2. **Poker (8 points)**
   Consider playing a simple poker game, for which one pays \$1 to play, draws 5 cards at random, and wins as follows if the hand is of a particular type:

Royal flush	\$250	(straight flush with ace as high card)
Straight flush	\$50	(five cards consecutively in order of the same suit, e.g., $4, 5, 6, 7, 8$ of hearts)
Four of a kind	\$25	(e.g., four threes of all suits, and the jack of spades)
Full house	\$9	(e.g., three cards of one kind, and two cards of another kind)
Flush	\$6	(all five cards of the same suit, e.g., 3, 6, 9, 10, jack of hearts)
Straight	\$4	(five cards consecutively in order of any suit, e.g., $3, 4, 5, 6, 7$ with 3 of hearts, 4 of spades, 5 of clubs, 6 of hearts, 7 of diamonds)
Two pairs	\$2	(two cards of one kind, and two cards of another kinds)
Pair of jacks or better	\$1	(e.g., two jacks, two queens, two kings, or two aces of any suit)

   Note that for a hand the highest type counts, e.g., three jacks and two kings are a full house and not two pairs. Also note that for a straight, or a straight flush, the ace may count as the high card (after the king), or as the low card (before the two). But wraparound, such as $king, ace, 2, 3, 4$ is not allowed.

   (a) (6 points) Compute the probability for each of the hand types listed above.
   (b) (2 points) Compute the expected win/loss for this game. Note that playing the game costs \$1, and hence \$1 has to be subtracted from each winning hand (e.g., the win for a straight flush is really \$49).

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3. **Independence (4 points)**
Two random variables X and Y on a sample space S are *independent* if

$$P(\{X = a\} \cap \{Y = b\}) = P(\{X = a\}) \cdot P(\{Y = b\}) ,$$

for all $a, b \in \mathbb{R}$, where the event $\{X = a\} = \{s \in S \mid X(s) = a\}$.

Let X and Y be the random variables that count the number of heads and the number of tails that come up when two coins are flipped. Show that X and Y are not independent.

4. **Bernoulli trials (4 points)**
Assume n independent Bernoulli trials are carried out with success probability p.

   (a) What is the probability of no successes?

   (b) What is the probability of at least one success?