CMPS/MATH 2170 Discrete Mathematics – Fall 14

11/5/14

8. Homework

Due Friday **11/14/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two.

1. **Counting (10 points)**
   (a) (5 points) How many strings of eight upper-case letters are there...
      i. ... that start with A, if no letter can be repeated?
      ii. ... that start and end with A, if letters can be repeated?
      iii. ... that start or end with A, if letters can be repeated?
      iv. ... that start with three vowels, if letters can be repeated?
      v. ... that contain at least three vowels, if no letter can be repeated?

   (b) (2 points) How many subsets of a set with 100 elements have more than one element?

   (c) (1 point) How many bijective functions are there between two sets of n elements?

   (d) (2 points) A bowl contains 10 red balls and 10 blue balls. You select balls at random without looking at them.
      i. How many balls do you have to select to be sure of having at least three balls of the same color?
      ii. How many balls do you have to select to be sure of having at least three blue balls?

2. **Permutations and Combinations (10 points)**
   Use combinations and permutations to solve the following problems. Clearly state how you model the problem: If you count ordered or unordered elements with or without repetition, and what kind of combination or permutation you use.

   (a) (2 points) How many bit strings of length 12 contain...
      i. ... exactly three 1s?
      ii. ... at least three 1s?

   (b) (2 points) How many permutations, without repetitions, of the letters $ABCDEFG$ contain the string FAB?

   (c) (2 points) How many strings can be made from the word $MISSISSIPPI$?

   (d) (4 points) A shop has the following macaroon flavors: almond, chocolate, pistacchio, strawberry, caramel, vanilla. How many different ways are there to choose...
      i. ... a dozen macaroons?
      ii. ... two dozen macaroons with at least two of each kind?

3. **Subsets and Binomial Coefficients (4 points)**
   Use a combinatorial argument to prove that $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, for all integers $0 \leq k \leq r \leq n$. Do **not** use plain algebraic manipulation.