CMPS/MATH 2170 Discrete Mathematics – Fall 14

10/22/14

6. Homework

Due Monday **11/3/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Recursive sequence (4 points)**
   Give a recursive definition of the sequence $\{a_n\}_{n\in\mathbb{N}}$ if
   (a) (2 point) $a_n = 1 + (-1)^n$
   (b) (2 point) $a_n = n^2$
2. **Recursive Definition (4 points)**
   Give a recursive definition of:
   (a) (2 points) the set of positive integer powers of 3.
   (b) (2 points) the set of polynomials with integer coefficients. *(I.e., for example* $4x^3 - 3x$ *or* $-3x^5 + 7x^2 + 4$*)*
3. **Fibonacci induction (5 points)**
   Let f_n be the n-th Fibonacci number. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Use a form of induction (which one are you using?) to prove that
   $$A^n = \begin{pmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{pmatrix}$$
   for all $n \in \mathbb{Z}^+$.
4. **Mod (4 points)**
   (a) (2 points) Evaluate these quantities:
   (i) $-17 \mod 2$, (ii) $144 \mod 7$, (iii) $-101 \mod 13$, (iv) $199 \mod 19$.
   (b) (2 points) List two negative integers and two positive integers that are contruent to 4 modulo 12.
5. **Division (6 points)**
   (a) (2 points) Prove or disprove that if $a|bc$, where a, b, c are positive integers, then $a|b$ or $a|c$.
   (b) (4 points) Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$.