CMPS/MATH 2170 Discrete Mathematics – Fall 14

10/15/14

5. Homework

Due **10/24/14** at the beginning of class

Remember, you are allowed to turn in homeworks in groups of two. One writeup, with two names.

1. **Cardinality (6 points)**
   Determine whether the sets below are finite, countably infinite, or uncountable.
   (a) (2 points) The negative even integers.
   (b) (2 points) The integers between 0 and 1000.
   (c) (2 points) The rational numbers between 0 and 1000.

2. **Weak induction in steps (7 points)**
   Let $P(n)$ be the following statement:
   $$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$
   The following sub-problems guide you through a proof by weak induction that $P(n)$ holds for all $n \in \mathbb{N}$.
   (a) (1 point) In order to understand the claim, verify it by hand for $n = 4$.
   (b) (1 point) What is the statement $P(1)$? Show that $P(1)$ is true, which completes the base case.
   (c) (1 point) What is the inductive hypothesis?
   (d) (1 point) What do you need to prove in the inductive step?
   (e) (3 points) Complete the inductive step.

3. **More weak induction (10 points)**
   Use weak induction on n to prove the following claims.
   (a) (5 points) Let $n \in \mathbb{N}$ and let $p_1, p_2, \ldots, p_n$ be propositions. Prove that $\neg(p_1 \wedge p_2 \wedge \ldots \wedge p_n)$ is equivalent to $\neg p_1 \vee \neg p_2 \vee \ldots \vee \neg p_n$.
   (b) (5 points) $n! < n^n$ for all integers $n \geq 2$.

4. **Strong induction (6 points)**
   Let $f(n)$ be the n-th Fibonacci number. Use strong induction to prove that for every positive integer n:
   $$f(n) = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$$
   (Hint: There need to be multiple base cases. For the inductive step, make sure to use the recursive definition of the Fibonacci numbers. And it might be useful to know that $\frac{3+\sqrt{5}}{2} = (\frac{1+\sqrt{5}}{2})^2$.)